OHMIC LOSSES IN COAXIAL GYROTRON CAVITIES WITH CORRUGATED INSERT

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Ohmic losses in coaxial gyrotron cavities with corrugated insert are calculated on the basis of two theories – the surface impedance model (SIM) and the singular integral equation (SIE). It is found that SIE predicts significantly lower losses in the insert of the resonator of the coaxial gyrotron for ITER operating in the $TE_{34,19}$ mode at 170 GHz.

As a consequence of this it is possible to increase the radius of the insert and to reduce mode competition.

Introduction

Coaxial cavity gyrotrons have the potential to generate microwave power in the multimegawatt range in continuous wave operation at frequencies around 170 GHz as required for ITER [1]. Various theoretical aspects of coaxial cavities with corrugated inner conductor have been discussed in the literature [2-7]. In these papers the insert is described by means of the surface impedance model (SIM). Limits of this model were discussed in [8, 9] where it was proposed to exploit a more powerful and rigorous method based on the singular integral equation (SIE) [10, 11].

In the present work we address a very important practical issue: ohmic losses in the corrugated insert. We show that SIE predicts significantly lower losses than SIM. Because of this, it is possible to increase the size of the insert and to improve the mode competition scenario.

General expressions for losses in gyrotron resonators

To characterize ohmic losses in the inner conductor of the coaxial gyrotron cavity (see, Fig. 1), we introduce the averaged densities of losses in the top, bottom, and side surfaces of corrugations and the density averaged over the corrugation period:

$$\rho_{in}^{top}(z) = \rho_0(z)\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{top} \qquad \rho_{in}^{bot}(z) = \rho_0(z)\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{bot}$$
$$\rho_{in}^{side}(z) = \rho_0(z)\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{side} \qquad \bar{\rho}_{in}(z) = \rho_0(z)\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{per} \tag{1}$$

where

$$\rho_0(z) = \rho(z)/\left|u(r_\perp)\right|^2, \qquad \rho(z) = \frac{1}{2}\delta k^2 \frac{|f(z)|^2}{\int_0^{z_{out}} |f(z)|^2 dz} \left|u(r_\perp)\right|^2 Q_{dif} P_{out}$$

and

$$\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{top} = \frac{2}{\varphi_S - \varphi_L} \int_{\varphi_L/2}^{\varphi_S/2} \left|u(r_\perp)\right|^2 \Big|_{r=R_{in}} d\varphi, \quad \left\langle \left|u(r_\perp)\right|^2 \right\rangle_{side} = \frac{1}{d} \int_{R_{in}-d}^{R_{in}} \left|u(r_\perp)\right|^2 \Big|_{\varphi=\varphi_L/2} dr$$
$$\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{bot} = \frac{1}{\varphi_L} \int_{-\varphi_L/2}^{\varphi_L/2} \left|u(r_\perp)\right|^2 \Big|_{r=R_{in}-d} d\varphi \tag{2}$$
$$\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{per} = \frac{1}{\varphi_S} \times \left((\varphi_S - \varphi_L)\left\langle \left|u(r_\perp)\right|^2 \right\rangle_{top} + \varphi_L \left\langle \left|u(r_\perp)\right|^2 \right\rangle_{bot} + \frac{2d}{R_{in}} \left\langle \left|u(r_\perp)\right|^2 \right\rangle_{side} \right)$$

Here δ is the skin depth, k is the free-space wave number $f(z)$ is the longitudinal RF field profile, $u(r_\perp)$ is the membrane function, Q_{dif} is the diffractive quality factor, P_{out} is the output power, $\varphi_S = 2\pi/N$, $\varphi_L = (L/S)\varphi_S$, L is the width, S is the period, d is the depth of corrugations, N is the number of corrugations, R_{cav} is the outer radius of the cavity, and R_{in} is the insert radius. Here $\varphi = 0$ corresponds to the middle of the corrugation.

Expressions (2) can be evaluated by means of either SIM or SIE using the corresponding formulas for the membrane function $u(r_\perp)$.

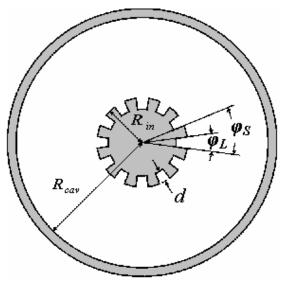

Fig. 1. The transverse cross-section of the coaxial gyrotron cavity.

Surface impedance model

SIM is based on the assumption that the width of the corrugations is smaller than half of the wavelength of oscillations ($L<\lambda/2$). In this approximation the fields can be assumed homogeneous inside corrugations, although they can vary from one corrugation to the next corrugation according to the azimuthal wave number. Here the corrugation is considered as a part of a rectangular waveguide, and the fields can be approximated by a part of the rectangular $TE_{1,0}$ mode. The resulting expressions for ohmic losses and their derivation within this formalism can be found in [2-7].

Singular integral equation

The local density of ohmic losses depends on the local field at the surface of the inner conductor. In its turn, the local field near the corrugated surface of the insert is significantly enriched by higher spatial harmonics, even if the applicability criterion for SIM ($L<\lambda/2$) holds. Therefore, SIM which takes into account only the fundamental spatial harmonic may lead to significant errors in evaluating the local density of ohmic losses and their averaged values. A rigorous full wave technique is based on reduction of the two-dimensional boundary value problem for the membrane function to a singular integral equation. This method was initially proposed in [12] and later successfully applied to rigorous analysis of various RF structures [13, 14]. The main advantage of this approach is the fact that SIE is derived for the total field without making any simplifying assumptions. The efficient and mathematically sound method of discrete vortices [15] can be used in numerical calculations. In what follows we give a brief derivation of SIE for calculating the membrane function for a coaxial gyrotron cavity.

In the region ($R_{in}<r<R_{cav}$) the membrane function $u(r,\varphi)$ can be expressed in terms of a superposition of spatial harmonics:

$$u \equiv u^{+} = \sum_{n=-\infty}^{\infty} A_n f_n(r)\exp(ik_n\varphi) \qquad (3)$$

where $f_n(r)=G_{k_n}(\chi,\chi/C,\chi r/R_{cav})$,

$G_\nu(a,b,c)=\left(J'_\nu(a)N_\nu(c)-N'_\nu(a)J_\nu(c)\right)/\left(J'_\nu(a)N'_\nu(b)-N'_\nu(a)J'_\nu(b)\right)$

$k_n=m+nk_S$ and $k_S=2\pi/\varphi_S=N$.

Due to quasi-periodicity of the membrane function with respect to φ, we can consider only the interval ($-\varphi_S/2,\varphi_S/2$). Inside the corrugations the membrane function can be expressed in terms of Fourier series:

$$u \equiv u^{-} = \sum_{n=0}^{\infty} X_n g_n(r)\cos\left(\xi_n(\varphi+\varphi_L/2)\right)$$

(4)

where $g_n(r)=G_{\xi_n}(\chi/C',\chi/C,\chi r/R_{cav})$ and $\xi_n=\pi n/\varphi_L$. Introducing the function $F(\varphi)=(R_{cav}/\chi)\partial u^{+}/\partial r\big|_{r=R_{in}}$, we obtain from (3) the expression

$$F(\varphi)=\sum_{n=-\infty}^{\infty} A_n\exp(ik_n\varphi) \qquad (5)$$

Matching u^{+} and u^{-} and their r-derivatives at the interface between the waveguide and the corrugation $\varphi\in(-\varphi_L/2,\varphi_L/2), r=R_{in}$, we obtain the relations

$$\sum_{n=-\infty}^{\infty} A_n f_n(R_{in})\exp(ik_n\varphi)=\sum_{n=0}^{\infty} X_n g_n(R_{in})\cos\left(\xi_n(\varphi+\varphi_L/2)\right) \qquad (6)$$

$$F(\varphi)=\sum_{n=0}^{\infty} X_n\cos\left(\xi_n(\varphi+\varphi_L/2)\right) \qquad (7)$$

Making inverse Fourier transforms of (5) and (7), we can express unknown amplitudes of spatial and Fourier harmonics in terms of integrals of $F(\varphi)$:

$$A_n=\frac{1}{\varphi_S}\int_{-\varphi_L/2}^{\varphi_L/2} F(\theta)\exp(-ik_n\theta)\,d\theta \qquad (8)$$

$$X_n=\frac{2\varepsilon_n}{\varphi_L}\int_{-\varphi_L/2}^{\varphi_L/2} F(\theta)\cos\left(\xi_n(\theta+\varphi_L/2)\right)d\theta \qquad (9)$$

where $\varepsilon_n=\begin{cases}1/2, & n=0,\\ 1, & n\neq 0.\end{cases}$

Substituting (8) and (9) into (6), we obtain the integral equation

$$\int_{-\varphi_L/2}^{\varphi_L/2} H(\varphi,\theta)F(\theta)\,d\theta=0,\quad -\varphi_L/2<\varphi<\varphi_L/2 \qquad (10)$$

where $H(\varphi,\theta)=G_1(\varphi-\theta)+G_2(\varphi-\theta)+G_2(\varphi+\theta+\varphi_L)$,

$G_1(x)=(1/\varphi_S)\sum_{n=-\infty}^{\infty} W_{k_n}(\chi,\chi/C)\exp(ik_n x)$,

$G_2(x)=-(1/\varphi_L)\sum_{n=0}^{\infty}\varepsilon_n W_{\xi_n}(\chi/C',\chi/C)\cos(\xi_n x)$, and

$W_\nu(a,b)=G_\nu(a,b,b)$.

It can be shown that the kernel $H(\varphi,\theta)$ of the integral equation (10) has a logarithmic singularity at $\varphi=\theta$. As is well-known the integral equation of the first kind with a logarithmic singularity is similar to the Fredholm equation of the first kind which is often extremely ill-conditioned. Therefore its direct numerical analysis is difficult. However, the integral equation (10) can be easily reduced to a singular integral equation with additional condition (integral equation with the

Cauchy-type singularity) for which effective direct numerical methods of solution have been elaborated [15]. Differentiating (10), we obtain the desired singular integral equation

$$\int_{-\varphi_L/2}^{\varphi_L/2} H'(\varphi,\theta)F(\theta)d\theta = 0, \quad -\varphi_L/2 < \varphi < \varphi_L/2 \qquad (11)$$

where the prime means differentiation with respect to φ . Integrating we find the necessary additional condition

$$\int_{-\varphi_L/2}^{\varphi_L/2} M(\theta)F(\theta)d\theta = 0 \qquad (12)$$

where $M(\theta) = \int_{-\varphi_L/2}^{\varphi_L/2} H(\varphi,\theta)d\varphi$

The singular integral equation (11) with additional condition (12) can be solved directly using different discretization schemes [10, 11, 13]. After numerical integration of (11) and (12) it is possible to compute with desired accuracy the local and averaged values of membrane functions which enter into the densities of ohmic losses (2).

Numerical example

The coaxial cavity gyrotron for ITER [1] operates in the $TE_{34,19}$ mode. Taking $\sigma = 5.7\cdot 10^4(\Omega\cdot mm)^{-1}$ which is the heat conductivity of ideal copper at room temperature and assuming that the generated RF power in the cavity is 2.2 MW, we calculated ohmic losses using the two theories (Table I).

Table I. Ohmic losses in the insert in the middle of the cavity where RF field is maximal.

density (kW/cm^2)	SIM	SIE
ρ_{in}^{top}	~0	0.009
ρ_{in}^{side}	0.024	0.010
ρ_{in}^{bot}	0.048	0.019
$\overline{\rho_{in}}$	0.057	0.027

It is evident that calculations based on SIE predict significantly lower losses in comparison with predictions of SIM. As an important practical consequence of this result it follows that R_{in} can be increased from its present value 8.0 mm to 8.2 mm without overheating the insert. Indeed within the SIM formalism such an increase results in $\overline{\rho_{in}} = 0.06\,kW/cm^2 \to 0.12\,kW/cm^2$ which can not be tolerated, while following SIE we obtain $\overline{\rho_{in}} = 0.03\,kW/cm^2 \to 0.06\,kW/cm^2$ which is well below the tolerable limit of $\overline{\rho_{in}} = 0.1\,kW/cm^2$. The thicker insert reduces mode competition, because quality factors of the modes with radial index 19 decrease only slightly (e.g., 1642→1606 for the operating $TE_{34,19}$ mode), while the relative decrease of quality factors of the parasitic modes with radial index 20 is much larger (e.g., 1297→1123 for the parasitic $TE_{31,20}$ mode). As a consequence the gyrotron is expected to oscillate only in the modes with radial index 19 with a broader region of oscillations of the operating mode.

Conclusions

The importance of inclusion of higher spatial harmonics in evaluating ohmic losses in a corrugated insert has been demonstrated. This can be achieved by means of the method of singular integral equation

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