TO THE THEORY OF CYCLOTRON MASER WITHOUT INVERSION (THE CYCLOTRON INSTABILITY IN THE NONRESONANT ELECTRON MEDIUM)

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The nonresonant regime of inversionless maser is investigated. The results of numerical calculations are presented that verify the analytical conclusions. The regime of enhances parametric instability due to the effect of “drawing” of waves frequency to the resonance is revealed.

Introduction.

This work is devoted to the investigation of novel regimes of cyclotron instability, realized in the form of parametric instability of two waves with close frequencies coupled in the modulated medium of charged particles. It is important that such ensembles of particles are stable against generation of the monochromatic radiation, so that these regimes can be named as “inversionless” by analogy with quantum effect of Lasing Without Inversion [1,2]. The first effect that was proposed [2] is the generation of cyclotron radiation by a modulated ensemble of resonant electrons; in this regime the amplification mechanism corresponds to the parametric interaction with modulated active susceptibility. Recently in papers [3,4] it was predicted that the amplification of bichromatic high-frequency radiation in the presence of low-frequency modulation of electrons is possible even if there is no energy exchange between waves and particles without modulation. Such a medium is reactive with respect to the monochromatic radiation - due to the absence of resonant particles. In this regime not scattering, but simultaneous amplification of two waves is realized.

The theoretical model

The regime of “nonresonant” instability can be realized for two waves with close frequencies propagating in the directions different from perpendicular with respect to the constant magnetic field. We consider here the most optimal scheme where two waves propagate along the magnetic field in opposite directions.

$$\mathbf{E}(z,t) = \mathrm{Re}\,\mathbf{e}_+ \sum_{j=1}^{2} E_j \exp\left(ik_j z - i\omega_j t\right) \qquad (1)$$

Here $\omega_1 \approx \omega_2$, $k_1 \approx -k_2$, $\mathbf{e}_+ = \mathbf{x}_0 + i\mathbf{y}_0$. Consider the interaction of this field with beam of electrons that are distant from the resonance with these two waves, so that cyclotron synchronism detunings $\Delta_{1,2}$ are large compared with the reversed time of interaction, where

$$\Delta_j = \omega_j - \frac{eB}{mc\gamma} - k_{\parallel j} V_\parallel = \Delta_{0j} - k_{\parallel j} V_\parallel, \;\; \Delta_{0j} = \omega_j - \frac{eB}{mc\gamma}. \qquad (2)$$

At the same time the fulfillment of parametric synchronism condition is required:

$$\omega_1 - \omega_2 \approx \left(k_1 - k_2\right) V_\parallel. \qquad (3)$$

The interaction of one of two waves with this electron ensemble results in no energy exchange between wave and electrons. The response of the medium to the action of one wave is reactive.

The main idea is the following. Since the partial synchronism detunings Δ_1 and Δ_2 (2) include the Doppler shifts of different sign, the modulation of electrons longitudinal velocity on time and longitudinal coordinate provides the oscillation of Δ_1 and Δ_2 in opposite phases. That results in oscillations of electrons reactive responses to these two waves in opposite phases. Then the equations of waves coupling are obtained in the following simplified form:

$$\begin{cases} \dot{E}_1 + i\chi_0 E_1 = -i\chi_M\, e^{i\varphi_M} E_2 \\ \dot{E}_2 + i\chi_0 E_2 = i\chi_M\, e^{-i\varphi_M} E_1 \end{cases} \qquad (4)$$

It is obvious that such system admits the unstable solution, the simultaneous amplification of two waves. The required type of modulation of electrons corresponds to the following form of the distribution function

$$f\left(p_\parallel, p_\perp\right) = f_0 + f_M \cos(\varphi_M + (k_1 - k_2)z - (\omega_1 - \omega_2)t), \qquad (5)$$

where $f_M\left(p_\parallel, p_\perp\right) = -f_M\left(-p_\parallel, p_\perp\right)$. Being settled initially (at t=0) such type of modulation is maintained during sufficiently long time period if the condition (3) is fulfilled.

The mechanism of energy exchange between two waves and nonresonant electrons can be explained in one-frequency frame, where $\omega_1 = \omega_2$, $k_1 = -k_2$. This frame is especially convenient because it helps to clarify the main differences between the regime of “nonresonant” instability and usual induced scattering process realized in similar conditions. The energy exchange between the field and medium is absent in the regime of scattering in this frame because the frequencies of incident and scattered waves are equal.

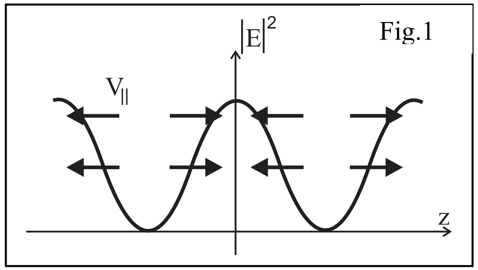

Fig.1

As it was shown in paper [4], the oscillatory motion of nonresonant relativistic particle in the field of standing wave (in the presence of constant magnetic filed) is accompanied by the change of its averaged energy:

$$\frac{d}{dt}\langle w\rangle = -\frac{e^2}{m\gamma_0}\frac{1}{2\omega\Delta_0}\left(1 - \frac{V_\perp^2}{c^2}\frac{\omega}{\Delta_0} + \frac{\omega}{\Delta_0}\left(2\frac{V_\perp^2}{c^2}\frac{\omega}{\Delta_0} - 1\right)\right)\left(V_\parallel \frac{\partial}{\partial Z}|E|^2\left(Z(t)\right)\right) \qquad (6)$$

The electron energy decreases if the particle moves in definite direction with respect to direction of the gradient of the field amplitude. Setting the dependence of particles longitudinal velocity to be periodic in definite correspondence with the standing wave at the initial moment (Fig.1) the decrease of the energy of the electron ensemble takes place.

The proposed in the paper [3] method of creation of the necessary modulation is obvious. The required type of modulation is produced in the laboratory frame (where the frequencies of two waves are different) as result of preliminary short-time action of the longitudinal wave with frequency $\Omega=\omega_1-\omega_2$ and wave number $\kappa=k_1-k_2$. But it is especially worth to note that the same type of modulation is created self-consistently under the action of two high-frequency interacted waves. It is the nonlinear source of modulation. For establishing this fact it is sufficient to trace the change of averaged longitudinal momentum under the action of two waves in quadratic approximation to the wave amplitude (see for details [4]):

$$\frac{d}{dt}P_{\parallel}=\frac{e^2}{m\gamma_0}\frac{1}{2\Delta_0^2}\frac{V_\perp^2}{c^2}\left(\frac{\partial}{\partial Z}|E|^2(Z)\right) \tag{7}$$

If the cyclotron detuning Δ_0 is positive the created periodic dependence of longitudinal momentum corresponds to the conditions of energy transfer from electrons to the waves.

The damage of initial modulation leads to the saturation of instability. The reasons for such damage can be the following. First of all it is the ballistic or dynamic transformation of initial modulation of longitudinal velocity to the modulation of space density. In this case the time of saturation can be estimated by following relation:

$$t_{sat}\sim 1/\Delta_M \tag{8}$$

where

$$\Delta_M\sim (k_1-k_2)\Delta V_{\parallel} \tag{9}$$

($\Delta V_{\parallel}$ -is the spread of longitudinal velocity in the electron ensemble), if the modulation of electrons is settled initially and nonlinear distortion of initial modulation is negligible; otherwise

$$\Delta_M\sim\frac{eEV_\perp\omega}{mc\gamma\Delta_0}. \tag{10}$$

Besides the damage of initial modulation can be caused by the destroying influence of excited longitudinal plasma field. In this case the saturation time is defined by plasma frequency ω_p of electron ensemble:

$$t_{sat}\sim 1/\omega_p\ . \tag{11}$$

In paper [4] the kinetic theory of this effect was developed in linear, quasi-vacuum approximations under condition $t<<t_{sat}$ when the saturation effects are negligible. The resulting equations of parametric coupling of two waves has the form:

$$\begin{cases}\dot{E}_1=-e^{i\varphi_M}\left(\Gamma t+iD\right)E_2,\\ \dot{E}_2=+e^{-i\varphi_M}\left(\Gamma t+iD\right)E_1,\end{cases} \tag{12}$$

where the coupling coefficients are defined by the formulas:

$$\Gamma=-\frac{2\pi e^2V_\perp^2\omega}{m^2\gamma^{\ 2}c^2}\int dp_\perp dp_\parallel f_M\,2kp_\parallel\frac{1}{\Delta_0^2}\ ,\qquad D=\frac{2\pi e^2V_\perp^2\omega}{m^2\gamma^{\ 2}c^2}\int dp_\perp dp_\parallel f_M\,2\,kp_\parallel\frac{1}{\Delta_0^3}\ . \tag{13}$$

One of two bicomponent exponential modes ($E_j\propto\exp\int_0^t\mu(\tau)d\tau$) with the ratio of partial amplitudes equal to $\left(E_1/E_2\right)=-i\left(D/|D|\right)\exp\left(i\varphi_M\right)$ has the following exponent:

$$\mu=\mu'+i\mu''=|D|-i\Gamma t\left(D/|D|\right). \tag{14}$$

This mode is unstable. It is important to note that due to the parametric interaction the frequency of amplified mode is drawing to the resonance with particles, although this frequency shift just as the increment must be small compared with the initial detuning from the resonance for the approximations of the analytical calculations to be fulfilled.

The results of numerical calculations

Here we present the results of numerical analysis of the “nonresonant” regime of parametric instability, fulfilled on the basis of strict equations of particles motion in the field of two electromagnetic waves with time-dependent amplitude, prescribed circular polarization and spatial structure, and the field of longitudinal plasma wave, excited during interaction. The waves amplitudes obey the strict wave equations. Both the examination of analytical results and analysis of the parametric regime out of the frame of the analytical approximations were done. Different scenarios of amplification process were revealed.

The first type of instability process development is realized if the electron density is sufficiently low, so that the following condition is fulfilled:

$$\omega_p,\mu'<<\Delta_M \tag{15}$$

The amplification scenario is well described analytically by equations (12) under this condition if the modulation of electron ensemble is formed preliminary and the initial wave amplitudes are relatively small (the nonlinear effects are negligible). On the Fig.2 the corresponding typical result of numerical calculation for the evolution of electrons energy is presented. The saturation of instability is defined by the expressions (8), (9) in this case. If the modulation of electron ensemble is formed under the nonlinear action of the two interacted waves but condition (15) is still fulfilled then the behavior of the system resembles the previous one with the substitution of parameter (9) by the nonlinear one (10). The corresponding numerical result is presented on the Fig.3. In both cases the efficiency of the amplification process (the ratio of the extracted electrons energy to the

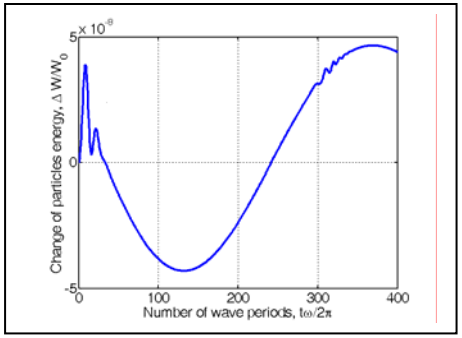

Fig.2

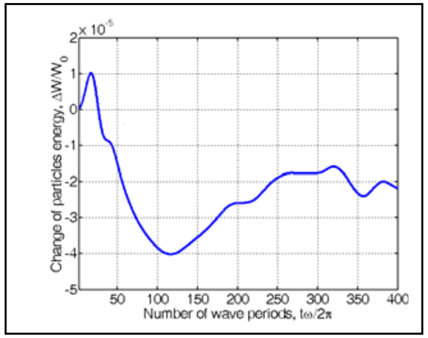

Fig.3

initial kinetic energy of electron ensemble) is very low. It can be estimated using the solution of equation (12) in the following way:

$$\frac{\Delta\langle W\rangle}{W_0} = -\left(\frac{e|E|}{mc\omega}\right)^2 \frac{V_\perp^2/c^2}{2\gamma_R(\gamma_R-1)}\left(\frac{\omega}{\Delta_0}\right)^3 \lesssim \frac{\Delta_M^2}{\omega\Delta_0} << 1 \qquad (16)$$

The gain coefficient (the ratio of the change of the waves intensity to the initial value) in this case is also small:

$$\frac{\Delta I}{I_0} = \frac{2\mu'}{\Delta_M} << 1 \qquad (17)$$

The much more effective instability process is realized if the other condition is fulfilled:

$$\mu' >> \Delta_M, \omega_p \qquad (18)$$

The typical scenario of the amplification process in this condition is presented on Figs.4,5.

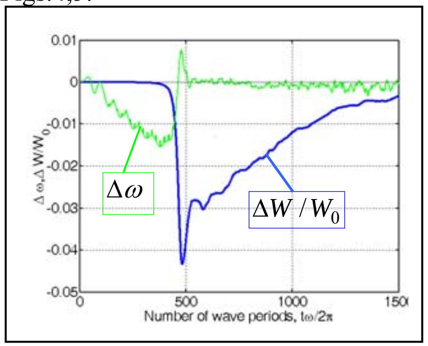

Fig.4

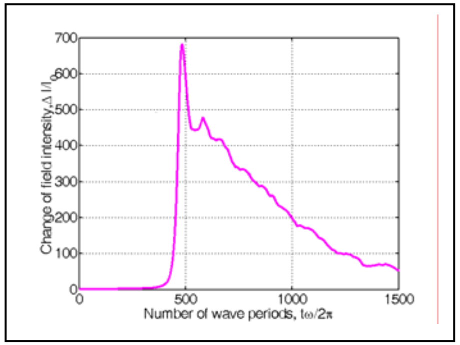

Fig.5

The extraction of kinetic energy and gain coefficient reach about 5% and 10^3 correspondingly. The physical mechanism of such enhanced parametric instability is the following. The relatively weak amplification of the bi-component mode caused by the parametric interaction with "nonresonant" modulated electron ensemble is accompanied by the drawing of the waves frequency to the resonance with particles (see solution (14)). If the density of electrons is not so small (the condition (18) is fulfilled) this "drawing" process passes faster than the saturation processes and leads to the essential strengthening of the parametric coupling that results in the considerable increase of the energy exchange rate. This process is saturated at the nonlinear stage of interaction. The efficiency and gain coefficients can be estimated by the following formulas:

$$\frac{\Delta\langle W\rangle}{W_0} \sim \frac{1}{\gamma}\left(\frac{2p_\perp\omega_p}{mc\omega}\right)^{2/3}, \quad \frac{\Delta I}{I_0} = \left(\frac{\omega_p^4 p_\perp}{\omega^4 mc}\right)^{2/3} \qquad (19)$$

The solution presented in the Figs.4,5 corresponds to the following parameters. The intensity of 1mm radiation interacting with the ensemble of relativistic electrons (relativistic gamma factor equal to 1.7) with density $7*10^9$cm^{-3} reaches in maximum 500 kW/cm^2. The averaged power per unit volume of energy extraction process comes to 50 kW/cm^3.

Conclusion

The demonstrated regime of instability of two waves in "nonresonant" medium of charged particles conflicts with the customary Manley-Rowe relation. This regime is realized in conditions, typical for induced scattering process. These contradictions are canceled by taking into account that demonstrated energy extraction process is limited in time of interaction. On the contrary the induced scattering is asymptotic process, characterized by finished bunching of electrons in definite phase with the beat wave. We have shown that before this bunching process finishes the essential energy exchange process between medium and the field can take place.

The general conclusion can be made: the regimes of electromagnetic field – medium interaction, agreed with the quantum conservation laws, are settled asymptotically. The time of this transitional process depends on a particular system. This work was supported by RFBR grants № 03-02-17176, №03-02-17234.

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