NONLINEAR THEORY OF AN ANNULAR ELECTRON BEAM AND EIGEN FLUTE MODES INTERACTION NEARBY ELECTRON CYCLOTRON FREQUENCY

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An investigation into phenomenon of nonlinear interaction between charged-particle beams and eigen modes of a plasma filled waveguide structures is important for different branches of plasma physics. Different electrodynamics' and electronics' aspects of charged particle beams interaction with the eigen modes of plasma-filled chambers were studied in [1, 2]. Results of theoretical research into beam-plasma interaction find a lot of applications in fusion experiments with beam-heated plasmas [3], as well. Restricted plasma volumes not only change the excitation dynamics of bulk waves but also provide conditions that are favorable for surface waves propagation [4]. So it has motivated the choice of the subject for our study. The paper is devoted to nonlinear theory of the interaction between annular charged-particle beams and eigen modes of cylindrical metal chamber filled by magnetized plasma in the frequency range nearby electron cyclotron frequency. These modes, which propagate in the azimuthal direction strictly transverse to a constant external axial magnetic field, are called surface flute modes (SFM) [5]. Beam-plasma instability of the SFM is investigated in the single-mode approximation here.

Formulation of the problem

Let's consider a cylindrical metal chamber of radius R_2 with a coaxial plasma column of smaller radius R_1. An annular electron beam moves in the gap between the plasma and the metal wall, the gaps' width is assumed to be sufficiently small, so that $R_2 - R_1 << R_2$. The constant external magnetic field $\vec{B}_0$ is supposed to be oriented along the symmetry axis of the cylinder that is $\vec{z}$ -axis. Electrical conductivity of the metal wall is assumed to be high enough to satisfy the boundary condition $E_\tau(R_2) = 0$ for tangential component of the SFMs' electric field. Considered wave-guiding structure is assumed to be uniform along the axial direction. The desired set of the differential equations describing the nonlinear stage of the beam-plasma interaction can be obtained from the hydrodynamic equations for the plasma, Maxwell's equations, and the beam particles motion equations. Since the beam density is much lower than the plasma density ($n_b << n_p$), one can neglect both the effect of the beam on the dispersion properties of SFM and the effect of the self-field of the beam on the waves electromagnetic field. The dissipative processes in the plasma are described by introducing the effective collision frequency ν into the SFM's dispersion relation.

In the cold plasma approximation, Maxwell's equations can be split into two independent subsets of equations by representing the dependence of the wave electromagnetic field on the time t and the azimuthal angle φ in the following form

$$E, H = f(r)\exp(im\varphi - i\omega t) \quad (1)$$

One of the subsets describes the SFM field with extraordinary polarization, the electric field of the SFM being perpendicular to the external magnetic field $\vec{B}_0$. For the magnetic component H_z of the SFM field, it is possible to obtain a second-order differential equation whose solution is expressed in terms of the modified Bessel functions:

$$\partial^2 H_z / \partial\xi^2 + \xi^{-1}\partial H_z / \partial\xi - \left(1 + m^2/\xi^2\right)H_z = 0, \quad (2)$$

where $\xi = kr\psi$, $k = \omega c^{-1}$, c is light velocity, $\psi^2 = \left(\varepsilon_2^2 - \varepsilon_1^2\right)\varepsilon_1^{-1}$, ε_1 and ε_2 are the elements of the dielectric tensor of a cold magnetized plasma. Components of the SFMs' electric field in the plasma region are described in terms of a superposition of the modified Bessel functions $I_n(y)$ and their derivatives.

In the vacuum region $R_1 < r < R_2$, where a low-density electron beam propagates, SFMs' field is expressed in terms of the first- $J_n(y)$ and second-order $N_n(y)$ Bessel functions. One can find the following differential equation for magnetic component of the waves:

$$\partial^2 H_z / \partial\zeta^2 + \zeta^{-1}\partial H_z / \partial\zeta - \left(1 - m^2/\zeta^2\right)H_z = F_b, \quad (3)$$

where $F_b = -\frac{4\pi}{\omega\zeta}\left[\frac{\partial\left(j_\varphi\zeta\right)}{\partial\zeta} - imj_r\right]$, $j_r = -|e|\sum_{i=1}^{N}\delta(r - r_i)\delta(\varphi - \varphi_i)\frac{\partial r}{\partial t}$, $\zeta = kr$,

$j_\varphi = -|e|\sum_{i=1}^{N} r\delta(r - r_i)\delta(\varphi - \varphi_i)\frac{\partial\varphi}{\partial t}$, $\delta(x - x_0)$ is the delta function. Components of the SFMs' are expressed in terms of Bessel functions, their derivatives and beams' current densities j_r, j_φ in the region. In order to derive the equations for the envelope and phase of the wave, we turn to the following boundary conditions: the tangential electric field of SFM vanishes at the metal wall and it has a break at the plasma-vacuum boundary due to presence of the beam.

Using the standard procedures (see, e.g., [4]), one can obtain equations for the envelope and phase of the eigen modes excited in the chamber by charged-particle beam:

$$\frac{dA}{dt} = -\frac{Az}{P}\,\mathrm{Im}\,D(\omega) - \frac{\alpha D_p}{NzPL}\sum_{i=1}^{N}\left[\frac{m}{w}\frac{dR_i}{dt}L_1(\zeta_i)\sin(m\varphi_i + \Theta - \omega t) + \right.$$

$$\left. + R_i^2 L_2(\zeta_i)\cos(m\varphi_i + \Theta - \omega t)d\varphi_i/dt\right], \qquad (4)$$

$$\frac{d\Theta}{dt} = -\frac{\alpha D_p}{NzPLA}\sum_{i=1}^{N}\left[R_i^2\frac{d\varphi_i}{dt}L_2(\zeta_i)\sin(m\varphi_i + \Theta - \omega t) - \right.$$

$$\left. - mw^{-1}L_1(\zeta_i)\sin(m\varphi_i + \Theta - \omega t)dR_i/dt\right], \qquad (5)$$

where $A = E_\varphi B_0^{-1}$ is the dimensionless wave amplitude, Θ is the wave phase, $\alpha = n_b n_p^{-1}$, $z = |\omega_e|\Omega_e^{-1}$, $w = \omega\,\Omega_e^{-1}$, $R_i = r_i\Omega_e c^{-1}$, $\zeta_i = kR_i$, ω_e and Ω_e are electron cyclotron and Langmuir frequencies,

$L = J_m(\zeta_1)N'_m(\zeta_2) - J'_m(\zeta_2)N_m(\zeta_1)$, $P = d[D_p + L_2(\zeta_1)/L]/dw$, $\zeta_1 = kR_1$,

$\zeta_2 = kR_2$, $L_1(\zeta_i) = J_m(\zeta_i)N'_m(\zeta_2) - J'_m(\zeta_2)N_m(\zeta_i)$,

$L_2(\zeta_i) = J'_m(\zeta_i)N'_m(\zeta_2) - J'_m(\zeta_2)N'_m(\zeta_i)$,

$$D_p = \frac{I'_m(\psi\zeta_1)}{\psi I_m(\psi\zeta_1)} + \frac{m\varepsilon_2}{\varepsilon_1\psi^2\zeta_1}, \qquad \mathrm{Im}\,D(\omega) \approx \frac{\nu}{2\omega}\left[\frac{I_{m+1}(\xi_1)}{\psi I_m(\xi_1)} + \frac{2|m|\omega^2}{\Omega_e^2\zeta_1}\right].$$

The equation of motion for the beam electrons can be conveniently written in terms of the electron momentum $\vec{p} = \gamma m_e\vec{\upsilon}$ (where γ is the relativistic factor):

$$\frac{d\vec{p}}{dt} = e(\vec{E}_0 + \vec{E}) + \frac{e}{c}\left[\vec{\upsilon}\times\left(\vec{H} + \vec{B}_0\right)\right]. \qquad (6)$$

Here one can substitute the obtained expressions of the SFM field calculated in the region $R_1 < r < R_2$.

Results of numerical investigations

The set of model equations (4 - 6) was solved by using a fourth-order Runge-Kutta method. The number of particles used to model an electron beam was N= 500, because, with larger numbers of particles, the final results were found to remain essentially the same. The interaction of the beam electrons with the plasma boundary and metal chamber wall was simulated using the mirror reflection model. Results of numerical simulations of the development of the SFM instability induced by their interaction with the annular beam are illustrated in Figs. 1-5. The initial wave amplitude, wave phase, and the radial momentum of all beam electrons were assumed to be $A = 10^{-3}$, $\Theta = 0$, and $V_i = 0$, respectively. The initial distribution of the beam electrons over the azimuthal angle in the range $0 \le \varphi \le 2\pi$ was chosen to be approximately uniform, with a small random deviation ($\Delta\varphi = \pm 1\%$). Initial angular momentum of the beam electrons has been supposed equal to zR_i, with a similar small random deviation from this value.

Conclusions

Two-dimensional set of model equations describing the evolution of the envelope of the wave field, the phases of SFMs, and the coordinates and momentums of the electrons of a low-density beam has been derived. We have numerically analyzed the effect of the waveguide and beam parameters on the development of the SFM instability.

It was found that waves with $m < 0$ have not been excited, and excitation of SFM is highly sensitive to the beam electron density. The lower the beam density, the smaller is the wave amplitude in the saturation stage of the instability and the longer is the time scale on which the instability saturates. A decrease in the ratio $|\omega_e|\Omega_e^{-1}$ also reduces the time scale on which the SFM amplitude increases from its initial value to the maximum. SFMs with larger azimuthal mode numbers m are excited at slower rates, but their amplitudes in the saturation stage of the instability are larger. This is explained by the fact that the larger the azimuthal mode number m, the higher the phase velocity of the SFM and, accordingly, the larger difference between the wave phase velocity and the beam velocity (just this difference determines how much of the beam energy is transferred to the wave).

The growth rates of the resonant dissipative instability of SFMs' are slower than those of the resonant beam-driven instability. Moreover, an increase in the effective collision frequency results in an additional slowing of the growth rates. By increasing the transverse dimensions of the beam, it is possible to increase the difference between the beam velocity and the wave phase velocity and, consequently, to achieve larger amplitudes of the wave envelope in the saturation stage of the dissipative instability. Presented results of the theoretical research can be useful for studying stability of electron flow in electronical devices and for investigation of such undesirable phenomenon as surface waves excitation in plasma periphery of fusion devices.

Acknowledgement

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References

[1] Kuzelev M. V., Rukhadze A. A. “Electrodynamics of Dense Electron Beams in a Plasma” (Nauka, Moscow, 1990).
[2] Miller R. B. “Introduction to the Physics of Intense Charged Particle Beams” (Plenum, New York, 1982).
[3] Fujiwara M., Komeko O., Komori A., et al. Plasma Physics and Controlled Fusion, 1999, **41**, 157.
[4] Kondratenko A. N., Kuklin V. M. “Fundamentals of Plasma Electronics” (Energoatomizdat, Moscow, 1988).
[5] Girka V., Puzyrkov S. Plasma Physics Reports, 2002, **28**, 351.

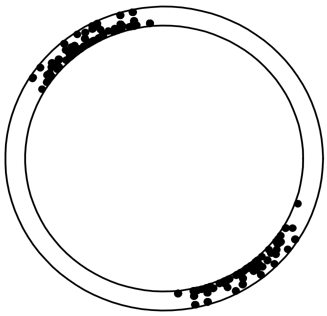

Fig. 1. Distribution of the beams’ particles in co-ordinate space (cross-section of the chamber) at the time of the SFMs’ instability saturation.

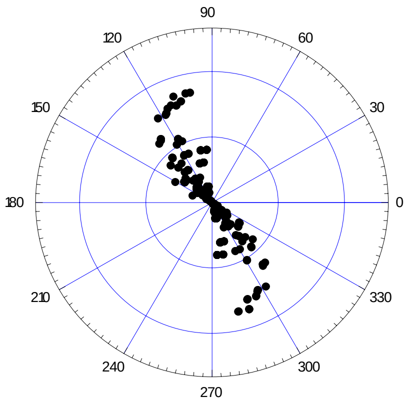

Fig. 2. Distribution of the beams’ particles in phase space (azimuthal angle vs azimuthal impulse) at the time of the SFMs’ instability saturation.

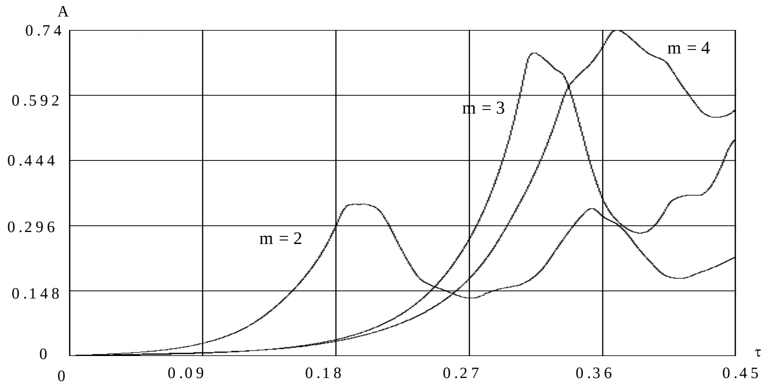

Fig. 3. Development of the SFMs' dissipative instability for different azimuthal mode numbers m , when $\nu = 0.1$.

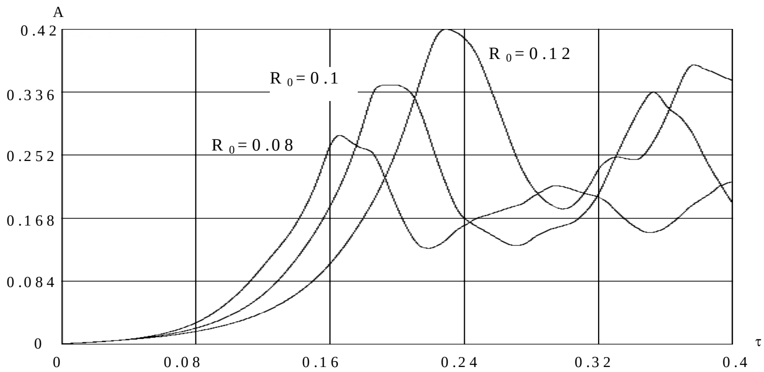

Fig. 4. Development of the SFMs' instability for different transverse dimensions of the beam region, when m =2 and $\nu = 0.1$.

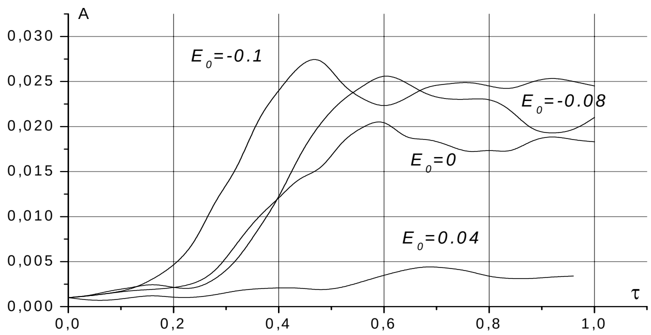

Fig. 5. Development of the beam-driven instability of SFM with m =2 for the cases of constant radial electric field application.