Relativistic downshift frequency effects on electron cyclotron emission measurement –Measurements of electron density in tokamak and electron temperature in LHD-

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Possibility of electron density measurement in a medium temperature tokamak plasma and the relativistic effect on the electron temperature measurement in LHD are investigated using a numerical calculation. The electron density measurement can be obtained by the observation of the frequency shift from non-relativistic electron cyclotron frequency in the tokamak. The electron temperature at the central region (r< 0.1) apparently decreases less than half even at 5 keV in LHD due to the its unique magnetic configuration.

I. INTRODUCTION

Plasma confined in a magnetic field radiates electron cyclotron emission (ECE)[1] with spectrum related to the local electron temperature. When the ECE spectrum is measured along the line of sight in a tokamak, the electron temperature profile along the path can be obtained[2-5]. In many tokamaks and stellarators[6-8], the measurement of ECE has been a useful diagnostic tool as the electron temperature.

To date electron temperature profiles has been derived from ECE using radii obtained from the radial dependence of the non-relativistic electron cyclotron (EC) frequency. When the electron temperature exceeds 5 keV, a relativistic down-shift becomes very large and the emissivity at the local position has a profile with finite extent in frequency. Such relativistic effects modify the one to one correspondence between the frequency and the spatial position in tokamaks, and the determination of the electron temperature profile from ECE becomes more complicated. The importance of the relativistic down-shift frequency effect has been well recognized in reactor-grade plasma[10-13]. The relativistic effect of the third harmonic ECE affects the second harmonic ECE which determined the spatial postion. It was also recognized the relativistic effect of the second harmonics on second harmonics which determined the spatial postion in the medium temperature ($2\ \mathrm{keV} \leq T_e \leq 10\ \mathrm{keV}$) tokamak plasma[14.15]. For the measurement from the low field side, the effect results in the radial shift of the electron temperature profile obtained using non-relativistic EC frequency apparently. The dependences of apparent radial shift on the plasma parameters are studied in the medium temperature plasmas in detail[15]. The shift depends on the electron density, temperature and magnetic structure. Since the shift depends on the electron density, there is a possibility of the electron density measurement if the electron temperature and magnetic structure are known. However, in the case of the measurement from low field side, the shift is very small. If the measurement from high field side is made, the shift will be large. So, here possibility of electron density measurement using relativistic down-shift frequency effect of ECE are presented using a numerical calculation. Altough the measurement of electron density was made by the observation of the optically thin harmonics[9], our case is using the optically thick harmonics.

On the other hand, the magnetic structure in the helical device is different from that in tokamak: the relation of frequency and spatial position is one-to-two correspondences, and the presence of maximum of magnetic field at the magnetic axis along the ECE sight line[8] at ϕ =-18 deg. poloidal plane. So, the effect of relativistic effect on ECE in helical devices is appeared to be different in comparison with tokamak. The emissivity at the magnetic axis is expected to decrease due to the relativistic effect and the decrement of emissivity will result in the apparent drop of electron temperature at the magnetic axis. However, there is little research on the relativistic effect of ECE in helical devices. The relativistic effect on electron temperature measurement in Large Helical Device (LHD) plasma has been evaluated by numerical calculations. The parametric dependences of the relativistic effect on electron temperature measurement are presented.

II. COMPUTATIONAL MODEL

The emission and absorption processes in a plasma are described by the radiation transfer equation[1]. The emissivity is calculated using the formula obtained by Trubnikov for the case of perpendicular propagation relative to a magnetic field in a tenuous plasma[16]. The electron velocity-distribution function is assumed to be a spherically-symmetric relativistic Maxwellian. The absorption coefficient is obtained from the emissivity applying Kirchhoff's law[1]. The details of this calculation method are in ref. 11 and 15. Note that the emissivity and absorption coefficient are proportional to electron density. When the electron density increases, the values of the emissivity and absorption coefficient increases, but the shapes of frequency dependence are not changed. The radial position of the electron temperature profile is determined by using the radius obtained from the radial dependence of the non-relativistic second harmonic EC frequency.

The plasma parameters used in the calculation are as follows: the major radius, *R*, are 3.4 m (tokamak) and 3.6 m (LHD), the minor radius, a, is 1 m (tokamak), the toroidal magnetic field, B_t, are 4.0 T (tokamak) and 2.75 T (LHD), and the profile of electron temperature, $T_e(r)$, is a parabolic function. The profile of electron density, n_e, is uniform or parabolic function.

III. ELECTRON DENSITY MEASUREMENT FROM ECE IN TOKAMAK [17]

We consider the measurement from the high field side observation. This is the propagation from outside to inside of the torus. The electron temperature profiles assumed and derived from the calculated second harmonic extraordinary mode ECE spectra in the case of $T_e(0)$=10 keV are shown in Fig. 1. The apparent radial displacement originates from the relativistic effect. When the electron density is increased, the apparent radial displacement is increased in the case of high field side observation.

The mechanism for relativistic effects on the measurement is briefly explained. A schematic diagram of the radial profile of the absorption coefficient or emissivity is shown in Fig. 2[14]. The frequency range of the absorption coefficient and emissivity expand due to the relativistic effect. The emission and absorption on the higher magnetic field side originate from electrons with higher energy for fixed frequency. When the emission at a given frequency (f_h) propagates to higher magnetic field side, electrons with higher energy emit absorb the emission. The balance between emission and absorption processes determines the radiance of the emission due to the relativistic effect. The effective resonance region for observed radiance is far from region of non-relativistic EC frequency. The frequency (f_0) is defined to be the non-relativistic second harmonic EC frequency that the true temperature ($T_e^{true}(f_0)$) at the frequency (f_0) equals the value of electron temperature ($T_e^{obsh}(f_h)$) observed at the frequency (f_h). The frequency shift (Δf) is defined to be $\Delta f = f_0 - f_h$ where $T_e^{obsh}(f_h) = T_e^{true}(f_0)$. The frequency difference (Δf) is related to the shape of emissivity and absorption coefficient and frequency difference depends on the electron temperature and density, magnetic field structure. So, the electron density can be estimated from the frequency difference if the electron temperature and magnetic structure is known.

We show the possibility of electron density measurement in following process by the numerical calculation. 1) The ECE radiance from high field observation is calculated assuming electron temperature and electron density. 2) The frequency shift is estimated from the calculated radiance. 3) The electron density is obtained using dependence of frequency shift on the electron density and the electron temperature. The electron temperature and magnetic structure are assumed to be known. Actually the electron temperature is obtained from the low field side observation.

Fig. 3 shows the dependence of frequency shift on the electron temperature in the case of uniform electron density. The frequency shift monotonically increases with electron density. The electron density profiles assumed and derived from the calculated ECE spectra in the case of $T_e(0)$=10 keV are shown in Fig. 4. The assumed profile is a parabolic square function. The electron density is estimated using the dependence interpolated by the Spline function. The agreement between assumed and obtained electron density is satisfactory.

Here, we discuss the constraint for the electron density measurement. First, it is necessary condition for the electron density measurement to measure the electron temperature profile exactly. The condition correponds to the optical thickness larger than 5, typically. Next, the relativistic effect of third harmonics does not affect the second harmonics, that is, the central electron temperature can be obtained. The possible region of electron density measurement is shown in Fig. 5. Moreover, the overlap between the second and third harmonics results in the constraint: $r/R > (2a/R-1) / 3+\Delta_{rel}$, where Δ_{rel} is apparent relativistic shift.

IV. ELECTRON TEMPERATURE MEASUREMENT IN LHD [18]

Since the magnetic field is bell-shaped along the ECE sight line in LHD, the relation between the frequency and the spatial position is one-to-two correspondences[8]. The magnetic field has maximum value at the magnetic axis in the R = 3.6 m configuration, the EC frequency at the maximum magnetic field is shifted to lower frequency due to the relativistic effect. Since ECE at non-relativistic EC frequency of the maximum magnetic field disappears in a high temperature, so electron temperature at the non-relativistic EC frequency of magnetic axis is expected to appear to drop.

The electron temperature profiles from ECE for various central electron temperature values in the case of $n_e = 1 \times 10^{19}$ m^{-3} is shown in Figure 6. The profiles are normalized to the true electron temperature. We found an apparent drop of electron temperature due to the relativistic effect at the magnetic axis and the electron temperature at the central region ($\rho < 0.1$) apparently decreases less than half even at 5 keV. Therefore it is necessary for precise measurement of the electron temperature profile to take into account the relativistic downshift frequency in LHD plasma.

The dependence of electron temperature from ECE on frequency for various electron density in the case of $T_e(0)$ = 5 keV is shown in Figure 7. The value of electron temperature is normalized by the true electron temperature. When the electron density is lower, the frequency shift from non-relativistic EC frequency is wider. The lower electron density means thinner optical thickness. That is, when the optical thickness is thinner, the frequency shift from non-relativistic EC frequency is wider.

V. SUMMARY

When the electron temperature exceeds 5 keV, the radial dependence of the EC frequency is modified due to the relativistic effects. We have studied the relativistic effects on ECE measurement in a tokamak and helical device using the numerical calculation taking into account of relativistic effect. The possibility of electron density measurement in a medium temperature tokamak plasma and the relativistic effects on the electron temperature measurement in LHD plasma are investigated .

It has been found that the electron density measurement can be evaluated by the observation of the frequency shift from non-relativistic EC frequency in tokamak. The frequency shift due to the relativistic effect is observed from high field side. The frequency shift depends on the electron temperature and density mainly. Since the electron temperature derived from observation of low field side, the electron density can be evaluated from the frequency shift.

The radiance at the magnetic axis decreases due to the relativistic effect in LHD, because the value of toroidal magnetic field has maximum at the magnetic axis along the ECE sight line. We found that the electron temperature at the central region ($\rho < 0.1$) apparently decreases less than half even at 5 keV, in the case of $n_e = 1 \times 10^{19}$ m^{-3}. Therefore it is necessary for precise measurement of the electron temperature profile to take into account of the relativistic effect in LHD plasma.

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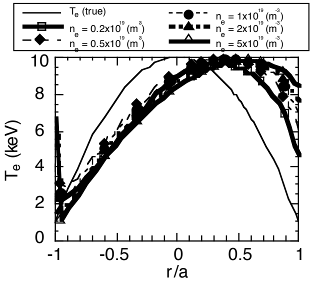

Fig.1: Temperature profiles assumed (solid line without mark) and derived (line with mark) from the high field side.

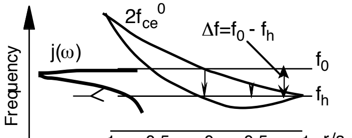

Fig.2: Schematic diagram of the radial profile of the absorption coefficient or emissivity

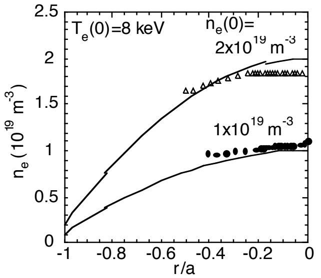

Fig. 5: Possible region of electron density measurement in the case of $B_t = 4$ T.

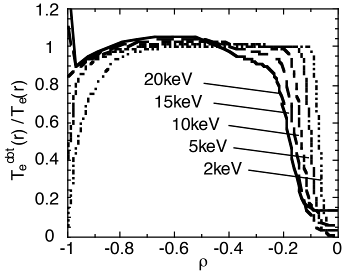

Fig.6: Electron temperature profiles for various electron temperature in the case of $n_e = 1 \times 10^{19}$ m^{-3}.

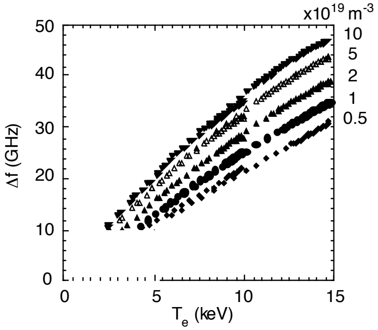

Fig. 3: Dependence of frequency shift on temperature for various electron density.

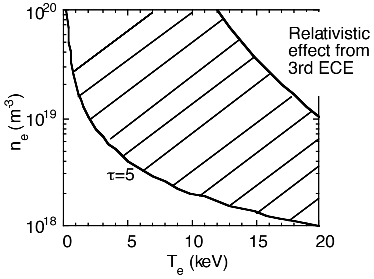

Fig. 4: Electron density profiles assumed (solid line) and derived (line with mark) from the calculated ECE spectra.

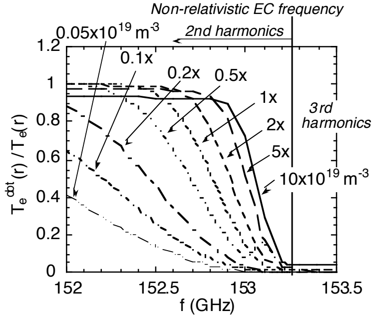

Fig. 7: Dependence of normalized electron temperature on frequency for various electron density in the case of $T_e(0)$=5 keV. Solid line represents the non-relativistic EC frequency at the magnetic axis.