Propagation and damping of electron Bernstein waves travelling from the high field side in tokamak plasmas

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1 Motivation

Propagation and ECR damping of EBWs in spherical tokamaks (ST) is usually analyzed assuming that absolute value of the tokamak magnetic field increases inward the plasma. In this case the perpendicular index of refraction $n_\perp$ grows as the wave approaches the ECR layer where the wave is fully absorbed regardless of the resonance harmonic number [1]. However, in addition to traditional regimes with monotonously increasing

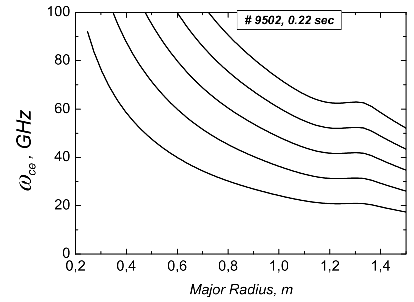

Figure 1: Total magnetic field $\boldsymbol{B}$ profile in MAST (V. Shevchenko et.al.).

$\boldsymbol{B}$, regimes with "magnetic wells" also occur in STs (fig.1). In case $\boldsymbol{B}$ at the well bottom is lower than at the plasma edge, there is a spatial region where access to the ECR for EBWs excited in the upper hybrid resonance (UHR) is open only from its high-field side. The B profile inversion modifies significantly the whole picture of the wave propagation and damping. Since the magnetic wells may become quite common with further improvement of ST performance, analysis of such configurations is of interest for assessment of EBW plasma heating an CD perspectives. In this report we consider basic features of the high-field side propagation and damping for the second cyclotron harmonic (which is now the lowest possible resonance harmonic) in a slab model.

2 EBWs in electrostatic approximation

Assume all plasma parameters depending on the single dimensionless coordinate x scaled in units of c/ω with ω being the wave frequency. Suppose that the magnetic field $\boldsymbol{B}$ is along the z - axis and both $\upsilon = \omega_{pe}^2/\omega^2$ and $q = \omega/\omega_{ce}$ grow inward the plasma. The wave frequency is chosen such that the fundamental resonance $q = 1$ is outside and the resonance $q = 2$ is inside the plasma with the upper hybrid resonance (UHR) located between them (fig.2). For such position of the (UHR) relative to the 2-nd ECR the

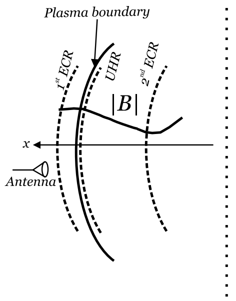

Figure 2: Model configuration.

υ profile steeper then the q profile is required that is the case in STs. We begin analysis of the EBW behavior assuming validity of the electrostatic approximation. Then the dispersion relation is

$$\varepsilon\left(n_{\perp}, n_{\|}, x\right) = 0, \tag{1}$$

where $\varepsilon \equiv \left(n_i \varepsilon_{ik} n_k\right)/n^2$ is the longitudinal dielectric function, ε_{ik} - the plasma dielectric tensor elements, $n_{\perp} = k_x c/\omega$ and $n_{\|} = k_{\|} c/\omega$. Outside the ECR layer, finite temperature effects enters the function ε through the parameter $\lambda = k_{\perp}^2 \rho_e^2 = \left(n_{\perp} q \beta\right)^2/2$, where ρ_e is the electron Larmor radius. As it will be seen, $n_{\perp}$ remains within the limits $1 \ll n_{\perp} \leq \beta^{-1}$, where $\beta = \nu_{te}/c$, $\nu_{te} = \sqrt{2T_e/m_e}$, in the whole region between the UHR and the $q = 2$ resonance, so that $\lambda \leq 1$ here. For qualitative investigation we expand ε in λ and keep only zero- and first-order terms. The electron Bernstein waves are produced via the linear conversion of incident electromagnetic waves with $n_{\|} \leq 1$, while applicability of the electrostatic approximation requires that $n_{\perp} \gg 1$. This permits one to omit terms proportional to $n_{\|}^2$ and $n_{\|}^4$ and obtain

$$\varepsilon_{xx} = \varepsilon_{xx}^{(c)} - \frac{1}{2}\upsilon\beta^2 \frac{3q^4}{\left(q^2-1\right)\left(q^2-4\right)} n_{\perp}^2 = 0. \tag{2}$$

where

$$\varepsilon_{xx}^{(c)} = 1 - \frac{\upsilon q^2}{q^2 - 1} \tag{3}$$

is the dielectric tensor element in the cold plasma. Solution to this equation is

$$n_\perp^2 = -\frac{2}{3\beta^2\upsilon}\varepsilon_{xx}^{(c)}\frac{\left(q^2-1\right)\left(4-q^2\right)}{q^4} \tag{4}$$

where $\varepsilon_{xx}^{(c)} < 0$ and $1 < q < 2$, shows that in the electrostatic approximation the EBW is confined between two cut-offs. One of them is the UHR, for which $\varepsilon_{xx}^{(c)}(x) = 0$, and the other one is the cyclotron resonance $q = 2$. The validity condition for the electrostatic approximation is $n^2 \gg |\varepsilon_{ik}|$. We consider here typical for STs high - density plasmas with $\upsilon \gg 1$, then the validity condition becomes $n_\perp^2 \gg \upsilon$. This condition breaks down close to the UHR and ECR. We do not consider first of these regions because analysis of wave behavior there is the subject of the mode coupling theory. In the dense plasma outside immediate UHR vicinity, Eq.(4) becomes

$$n_\perp^2 = \frac{1}{\beta^2}\frac{2\left(4-q^2\right)}{3q^2} \tag{5}$$

so that the characteristic value of $n_\perp$ is β^{-1}. The electrostatic applicability conditions is violated close to the $q = 2$ resonance where $n_\perp$ goes down requiring a full - wave treatment.

3 Full-wave equation near $q = 2$ resonance

To obtain a traceable full - wave hot plasma dispersion relation for EBWs with $n_\perp \sim n_\parallel$ and $n_\perp \ll \beta^{-1}$ near the $q = 2$ resonance we make some simplifications. Consider first the dielectric tensor elements ε_{ik}. It is well known that the elements can be presented as an infinite sums over cyclotron harmonic number $s, -\infty \ll s \ll \infty$ with each term of the sum related to the $q = s$. Since now parameter λ is small, we calculate the resonance ($s = 2$) terms of ε_{ik} up to the first order in λ using the zero order (cold plasma) approximation for non - resonant terms. In this approximation, the elements $\varepsilon_{xz}, \varepsilon_{zx}, \varepsilon_{yz}, \varepsilon_{zy}$ vanish, $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_\perp$, $\varepsilon_{xy} = -\varepsilon_{yx} = -ig$ and

$$\varepsilon_\perp = \varepsilon_\perp^{(c)} + \frac{1}{2}\frac{\lambda\upsilon}{n_\parallel\beta}Z\left(\xi\right),\ g = g^{(c)} + \frac{1}{2}\frac{\lambda\upsilon}{n_\parallel\beta}Z\left(\xi\right), \tag{6}$$

here $g^{(c)} = -\upsilon q/\left(q^2-1\right)$, Z is the plasma dispersion function defined according to [2] with the argument $\xi = (q-2)/qn_\parallel\beta$. The dispersion relation for EBWs obtain in the form

$$n_\perp^2 \varepsilon_\perp - \frac{\upsilon^2 q^2}{q^2-1}\left(1 - \frac{n_\perp^2 \beta (q+1) q}{2n_\parallel} Z(\xi)\right) = 0 \tag{7}$$

omitting E_z due to an absolute values of dielectric tensor elements here are of order $\upsilon \gg 1$. This equation is valid at $n_\perp < \upsilon$ and the electrostatic equation (4) is valid at $n_\perp^2 > \upsilon$. Thus, in the vicinity of the $q = 2$ resonance, where $n_\perp(x)$ goes down with x, there is a spatial region $\upsilon^2 > n_\perp^2 > \upsilon$ where Eqs.(4) and (7) are valid simultaneously. Eq.(7) represents an approximate dispersion relation valid at $\beta < n_\parallel \leq 1$ in the whole spatial region located between $q = 2$ resonance and immediate vicinity of the UHR. The necessary condition for this relation applicability is high plasma density, $\upsilon \gg 1$ at $q = 2$. The dispersion relation (7) can be also used in the "relativistic" case $n_\parallel \leq \beta$ if the "non-relativistic" plasma dispersion function Z is replaced by a proper relativistic dispersion function.

4 Wave behavior in the 2^{nd} ECR layer

Close to the 2^{nd} ECR layer the approximate dispersion relation is given by the second term of Eq.(7). Putting $(q-2)/q = x/l$, where l is the dimensionless characteristic scale-length of the magnetic field variation, the dispersion relation can be now written

$$n_\perp^2 = \frac{2n_\parallel}{3\beta Z(\xi)}, \tag{8}$$

where $\xi = x/L$ and $L = l\beta n_\parallel$ is the width of the resonance layer. Using the asymptotic expression $Z \sim -1/\xi$ at $|\xi| \gg 1$ find that at negative x outside the resonance layer Eq.(8) differs from the electrostatic equation (4) only by a constant factor (equal to 4) in the right-hand side. Inside the layer at $|\xi| \sim 1$ the function Z is a complex function with $|Z| \sim 1$ and $Re(Z) \sim Im(Z)$. Solution to Eq.(8) is also complex with $|n_\perp| \sim \left(n_\parallel/\beta\right)^{1/2}$, so that the applicability condition for the WKB theory in the resonance region takes the form $n_\perp L = l n_\parallel^{3/2} \beta^{1/2} \gg 1$. The estimation for the relativistic case can be obtained from this equation by putting $n_\parallel \sim \beta$. We assume that the WKB approximation is applicable in the region outside the resonance layer where $|n_\perp|$ grows. Suppose now that WKB is valid and consider solution to Eq.(9) inside the resonance layer. These solutions in a parameter free form are

$$N = \pm Z^{-1/2}, N = n_\perp \left(3\beta/n_\parallel\right)^{1/2}. \tag{9}$$

Brunches marked by signs $(-)$ and $(+)$ in Eq.(9), respectively, correspond to incident and reflected from the ECR waves. They are not coupled. Three important conclusions can be drawn from this analysis. First, incoming waves incident on the ECR layer from the high-field side are not converted in the resonance region into outgoing EBWs with large $n_\perp$ propagating on the low field side of the ECR. Instead, the incident waves become non-propagating beyond the resonance layer. Second, two effects are simultaneously responsible for $Im\,(n_\perp)$: the ECR damping and the wave transition into the non-propagation region. These two contribution can not be separated. Finally, since the function $|Z(\xi)^{-1}|$ has no zeros at finite $|\xi|$, two brunches of the dispersion curves are separated in the whole complex x plane. As a result, reflection from the ECR layer can only be due to approximate nature of the WKB theory.

5 Reflection from ECR layer

We analyze reflection of EBWs from the $q = 2$ cyclotron resonance with the use of the model wave equation obtained from the dispersion relation (9) by replacement $n_\perp \to -id/dx$:

$$U^{''}(x) + n_\perp(x)^2 U(x) = 0. \qquad (10)$$

Here $n_\perp^2$ is given by the right - hand side of Eq.(9) and we assume that function U is related linearly to the wave field components. More definite interpretation of this function is not required. In the WKB approximation Eq.(10) has two linearly independent solutions

$$U_1 = \frac{1}{n_\perp^{1/2}} \exp\left(-i\int_a^x n_\perp dx\right),\; U_2 = \frac{1}{n_\perp^{1/2}} \exp\left(i\int_a^x n_\perp dx\right) \qquad (11)$$

where a is an arbitrary constant. As it has been mentioned before, U_1 and U_2 represent incoming and outgoing waves, respectively. The solution to Eq.(10) describing the ECR damping and reflection of the wave incident from the high-field side vanishes at $|x| \gg L$ and has the asymptotic form

$$U = U_1 + RU_2 \qquad (12)$$

In this case the reflection coefficient R can be calculated as:

$$R = \frac{1}{\Gamma_0}\int_{-\infty}^{\infty}\left(Z^{1/4}\right)^{''} Z^{1/4} \exp\left(-i\Gamma_0 \int_{-\infty}^{\xi} Z^{-1/2}\,(\zeta) d\zeta\right) d\xi, \qquad (13)$$

where the prime denotes differentiation with respect to ξ and $\Gamma_0 = 2l\beta^{1/2} n_\parallel^{3/2}$. For Eq.(13) validity, Γ_0 must be large compared to unity.

Numerical evaluation shows that $|A|^2$ is negligibly small at $\Gamma_0 \geq 1$.
In the limiting case $|k_\perp| L < 1$ opposite to the WKB one, the reflection coefficient can be found analytically. In the present case the reflection coefficient is close to unity. Calculating the correction to it in the lowest order in $\gamma = 12^{-1/3}\Gamma_0^{2/3} \ll 1$ yields

$$|A|^2 = 1 - \alpha\Gamma_0^{4/3}, \alpha = 2\pi^2 Ai^2(0)/12^{2/3}, \tag{14}$$

where Ai is the Airy function.

6 Summary

1. Existence of EBWs in the region between the UHR and ECR requires inhomogeneous plasma density. Waves in this region are adequately described by the approximate full-wave dispersion relation (7).
2. Incoming waves incident on the ECR layer from the high-field side are not converted in the resonance region into outgoing EBWs propagating on the low field side of the ECR. Instead, the incident waves become non-propagating beyond the resonance layer.
3. In the WKB approximation, the waves are fully damped in the ECR layer. Reflection from the ECR layer is only due to approximate nature of the WKB theory.

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References

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