OPEN QUESTIONS IN ELECTRON CYCLOTRON WAVE THEORY

E. Westerhof,

FOM-Institute for Plasma Physics Rijnhuizen, Association EURATOM-FOM, Trilateral Euregio Cluster, The Netherlands, `www.rijnh.nl`

Abstract

Starting from a number of recently obtained results, this paper identifies some of the gaps in our theoretical description of the propagation and absorption of electron cyclotron waves as applied for heating and current drive. Open issues are identified in the linear theory of wave propagation in homogeneous and inhomogeneous plasma, in the quasi-linear theory of the corresponding plasma response, as well as in the nonlinear theory of wave absorption and wave propagation.

1 Introduction

This paper is not a review of our knowledge of electron cyclotron wave theory: excellent and comprehensive reviews exist discussing both theory and applications to fusion plasmas [1, 2]. Rather, this paper intends to present an overview of our current ignorance. In spite of the fact that the theory of electron cyclotron waves is said to be well established, a surprising number of question remains open at all hierarchical levels of theory ranging from linear theory in homogeneous plasma to the nonlinear theory in inhomogeneous plasma. Where appropriate recent advances in the theory are discussed. Inescapably the paper reflects the ignorance of the author rather than that of his colleagues, in particular, in the issues that are not covered. The discussion will be limited to the theory of the waves, their absorption, and the current that is generated in the plasma. The myriad of open issues on the plasma dynamics in response to the local electron cyclotron resonance heating (ECRH) and electron cyclotron current drive (ECCD) will not be discussed.

The paper is structured as follows. Section 2 deals with linear theory of electron cyclotron waves. First the wave power balance in a homogeneous plasma is discussed. Second, a number of questions in ray- and beam-tracing are raised. Section 3 discusses the quasi-linear plasma response. The main themes discussed, are methods for the incorporation of neoclassical effects in the model (in particular, those responsible for the bootstrap current) and self-consistent modeling of the current density evolution upon noninductive RF current drive. Section 4 treats two topics in nonlinear theory with recent progress: nonlinear absorption and electromagnetically induced transparency. The final Section 5 contains a brief summary.

2 Linear theory

2.1 Homogeneous plasma

In the linear theory of electron cyclotron waves there are actually a surprisingly number of open questions and some recent progress. Even on the point of something as basic as the wave propagation in a homogeneous plasma our knowledge is still incomplete. The reason for these and many of the following open questions lies in the large anti-Hermitian elements, ε_{ij}^{aH}, of the dielectric tensor which originate from the resonant plasma response. Even though the anti-Hermitian part of the dielectric tensor is large, wave absorption remains weak in the sense that generally the length (time) over which the wave is absorbed, is much larger than a wavelength (wave period). This is due to the modification of the wave polarization by the plasma response itself, such that the projection of the active component of the plasma current on the wave electric field remains small [6, 7],

$$\frac{|E_i^* \varepsilon_{ij}^{aH} E_j|}{|E_i^* \varepsilon_{ij}^{H} E_j|} \ll 1. \tag{1}$$

It has long been recognized [6] that a large anti-Hermitian part is incompatible with the assumptions leading to the wave power balance as it is usually obtained by averaging of the Poynting theorem over a wave period [3–5]. However, a generalization of the wave power balance has recently been obtained, which takes the usual form:

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} + Q = 0, \tag{2}$$

but where

$$\begin{aligned} W &= |A|^2 \frac{1}{16\pi\omega_0} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon_{ij}^H e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0} = -|A|^2 \frac{c^2}{16\pi\omega_0} \frac{\partial}{\partial \omega} (D_{ij}^H e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0}, \\ \mathbf{S} &= |A|^2 \frac{c^2}{16\pi\omega_0} \frac{\partial}{\partial \mathbf{k}} (D_{ij}^H e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0}, \\ Q &= |A|^2 \frac{\mathrm{i}c^2}{8\pi\omega_0} \left(D_{ij}^{aH} e_i^* e_j \right)\Big|_{\vec{\Gamma}=\vec{\Gamma}_0 + (\partial\phi/\partial\vec{\xi})}. \end{aligned} \tag{3}$$

In case the anti-Hermitian part of the dielectric tensor is negligible and consequently $D_{ij}^H e_j = 0$, these expressions reduce to the usual wave power balance found in the text books [3–5]. Whereas Q represents the true dissipated wave power, which can be found back in the rate of change of kinetic energy of the plasma due to the waves [5], the identification of W with the wave energy density and $\mathbf{S}$ with the wave energy flux is only phenomenological. Especially W can be seen to become negative in regions of resonant absorption where the wave dispersion becomes anomalous. The questions to the proper wave energy density and wave energy flux in these regions of resonant absorption remain wide open. Determination of the latter is only possible on the basis of 'microscopic models' of the medium [13].

In spite of these problems with the interpretation of the different terms, as shown in [9, 11], the 'power balance equation' (2,3) does give the correct description of the evolution of the wave intensity $|A|^2$, when written in the form

$$- \left.\frac{\partial\,\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}}{\partial\omega}\right|_{\vec{\Gamma}=\vec{\Gamma}_0} \frac{\partial|A|^2}{\partial t} + \left.\frac{\partial\,\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}}{\partial\mathbf{k}}\right|_{\vec{\Gamma}=\vec{\Gamma}_0} \cdot \nabla|A|^2 = 2\,\mathcal{I}\mathrm{m}\,\lambda^{\mathrm{mode}}\big|_{\vec{\Gamma}=\vec{\Gamma}_0+(\partial\phi/\partial\vec{\xi})}\,|A|^2. \quad (4)$$

Here, $\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}$ is the real part of that eigenvalue of the dispersion tensor that corresponds to the wave mode under consideration. The final term clearly again represents the power lost to the medium, where $\mathcal{I}\mathrm{m}\,\lambda^{\mathrm{mode}}$ is easily identified with the relevant element of ε^{aH} in the appropriate diagonal form of the dispersion tensor.

2.2 Inhomogeneous plasma

The next complication in the description of wave propagation and absorption comes from the introduction of inhomogeneity of the plasma medium. Because of the small wave lengths (λ) of electron cyclotron waves compared to typical scale lengths of the plasma equilibrium (major radius R and minor radius a with $R, a \gg \lambda$, the plasma inhomogeneity can be treated with the tested ray-tracing techniques of geometric optics. Generally one assumes that wave propagation in the context of ray-tracing can be described well using the cold plasma dispersion relation: the deviations to the direction of wave propagation stemming from warm plasma effects are assumed to be small. Wave absorption along this 'cold plasma' trajectory is then calculated using the warm plasma dispersion in some approximate (non- or weakly relativistic) form or in its full relativistic form. The TORAY code is one example of a numerical code in which these ray-tracing and absorption calculations have been implemented [15–17].

The assumption that warm plasma effects have negligible effects on the direction of wave propagation, however, is far from being the truth. In actual fact, the direction of wave propagation in the areas of resonant wave absorption can differ dramatically from those of cold plasma dispersion [14]. Only because the resonance regions are relatively narrow and wave propagation often is almost perpendicular to the resonance, are the integrated effects on the ray trajectories small. As noted in the previous paragraph the direction of wave propagation (i.e. the trajectory followed by the maximum of the wave intensity) is consistent with the direction of the 'wave energy flux'. The proper ray-trajectories for the warm plasma case are thus obtained from the common ray-tracing equations, using $\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}$ as the ray-Hamiltonian

$$\frac{d\mathbf{r}}{d\tau} = \frac{\partial\,\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}}{\partial\mathbf{k}} \qquad \frac{d\mathbf{k}}{d\tau} = -\frac{\partial\,\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}}{\partial\mathbf{r}} \quad (5)$$

In applications of ECRH and ECCD one is not dealing with the propagation in time of short wave pulses, but rather with the trajectory of a (on the time scale of propagation through the plasma) stationary wave beam. Consequently, only the spatial trajectories as described by (5) are relevant and complications stemming from the anomalous behaviour of the group velocity can be avoided.

As is well known, ray-tracing cannot provide a proper description of the evolution of a wave beam near its focus. The latter requires a proper account of wave diffraction. Like the wavelength, also the width of the beam w is generally small compared to length scales on which the equilibrium varies. This allows the use of the complex eikonal method [18–21] with the formal ordering

$$\lambda/L = \mathcal{O}(\kappa) \ll w/L = \mathcal{O}(\kappa)^{1/2} \ll 1 \qquad L = a, R \tag{6}$$

For a lowest order Gaussian beam this results in a set of equations existing of the usual ray-equation from geometric optics for the central ray of the beam supplemented with ordinary differential equations for the beam width and the phase front curvature. These have been implemented in the beam tracing code TORBEAM [22]. As in the case of most ray-tracing codes TORBEAM uses the cold plasma dispersion to calculate both the central ray-trajectory as well as the evolution of the beam width w and phase front curvature R_c. Warm plasma effects on beam propagation can straightforwardly be implemented in the code by substitution of the cold plasma dispersion relation with $\mathcal{R}\mathrm{e}\,\lambda^{mode}$. However, the ordering (6) underlying the beam-tracing equations might well fail in the narrow resonance layer ΔR_{res}, within which the wave dispersion properties can be strongly varying. In fact, near the resonance the ordering (6 must be supplemented with $L = \Delta R_{\mathrm{res}}$.

The possible break down of the ordering is also reflected by a consideration of the power absorption. Formally, the smallness of w compared with the scale lengths of the equilibrium (actually, the scale lengths over which the wave properties vary) implies that the absorption of the beam can correctly be described in terms of the absorption along the central ray itself. The effect of the finite beam width and phase front curvature on the absorption are assumed to be negligible. However, the finite time in which a resonant electron passes through the wave beam leads to a broadening of the resonance in velocity space [23, 24]:

$$\delta(\gamma - n\omega_{ce}/\omega - N_\parallel p_\parallel/m_e c) \rightarrow \sqrt{\frac{\pi}{2\Delta Q}}\exp\left[-\frac{(\gamma - n\omega_{ce}/\omega - N_\parallel p_\parallel/m_e c)^2}{2\Delta Q}\right] \tag{7}$$

with

$$\Delta Q = \left(\frac{\gamma v_\parallel}{\omega w}\right)^2 + \left(\frac{\gamma \Delta N_\parallel v_\parallel}{2c}\right)^2$$

where $\Delta N_\parallel = w/R_c$ is the effective spread in parallel wave vector over the cross-section of the beam due to the phase front curvature. Note that the resonance broadening in identical $\Delta Q = (\gamma v_\parallel/\omega w_0)^2$ both in the far field limit, where the beam width is large and the first term is negligible, as well as in the beam focus where $\Delta N_\parallel = 0$ and $w = w_0$ [24]. This is not surprising as one realizes that in both limits, one deals with a beam composed of the same Fourier modes in parallel refractive index. This resonance broadening can affect both location and width of the region of ECRH power deposition [24].

Not only do beam width and phase front curvature affect the absorption, vice versa absorption can affect the shape of the wave beam. In particular when a beam propagates oblique with respect to the resonance plane the wave dispersion and absorption can vary significantly over the beam cross-section. This is of course related to the failure of the condition $w/\Delta R_{\mathrm{res}} \ll 1$, and requires a revision of the eikonal method in the light of this failure. Also asymmetries in the (parallel) electron velocity distribution, in particular under conditions of strong current drive or electron run-away could affect the beam evolution. In this case absorption from the wave beam would become inhomogeneous in Fourier space (parallel wave vector) rather than in real space. Both effects call for methods that allow for a variable and non-Gaussian transverse structure of the wave beam.

2.3 Electron Bernstein waves

As applications of electron Bernstein waves (EBW) are relatively recent, also the theoretical basis of EBW propagation and absorption is less developed. Moreover, the excitation of EBW is a problem in itself. The basic method is conversion of slow X-mode to EBW at the Upper Hybrid resonance. Slow X-mode waves launched directly from the high field side will first cross the cyclotron resonance before reaching the Upper Hybrid. Thus, only when the slow X-mode is is weakly absorbed (as in case of nearly perpendicular propagation) does this lead to efficient EBW excitation. For low field side launch two schemes can be discriminated. The one being direct conversion of X-mode to EBW in the edge of the plasma. This requires a plasma with a relatively steep density gradient such that the fast X-mode cut-off and the Upper Hybrid resonance are close enough to allow efficient tunneling between the two [25]. The other scheme uses conversion of O-mode to the slow X-mode which is subsequently converted to EBW at the Upper Hybrid layer [26, 27]. The O- to slow X-mode conversion requires that the respective cut-offs of the two modes are very close. This translates into a critical value for the parallel refractive index around which efficient mode conversion occurs.

Although ray-tracing codes for EBW are existing, the extension of the beam tracing codes to the EBW will be of interest. In particular the question how an O- or X-mode wave beam converts to an EBW beam is of interest, but not trivial. What happens to beam focus when it lies beyond the point of mode conversion? Or, how best to focus an EBW beam? Most questions raised above on beam propagation and absorption apply equally to propagation of EBW beams.

Presently one often uses a non-relativistic dispersion relation to describe the propagation of EBW. Yet, experience from ECRH shows that relativistic effects are often essential. There is no principal obstacle to the use of relativistic dispersion in the calculation of EBW propagation and absorption. Recent calculations [28] show that relativistic corrections indeed are important.

3 Quasi-linear theory

Conservation of energy implies that the energy absorbed from the wave by the plasma is recovered in the response of the plasma as an increase in plasma energy. The increase in plasma energy consistent with the absorbed power Q as given by the wave power balance equation (2) is provided by the quasi-linear plasma response [29], which describes the particle diffusion in phase space as a consequence of the interaction with the waves.

3.1 Quasi-linear Fokker-Planck model including neoclassical effects

On a collisional time scale, the ECRH driven quasi-linear diffusion is balanced by the effects of Coulomb collisions a process that is described by the quasi-linear Fokker-Planck equation. The latter is usually treated in the long mean free path limit or so-called banana regime. In this regime the quasi-linear Fokker-Planck equation can be averaged over the orbits of the particles to obtain an orbit averaged quasi-linear Fokker-Planck equation. In terms of the so-called low field side coordinates $\mathbf{I}_0 = (r_0, p_0, \theta_0)$ (minor radius, momentum, and pitch angle), which are invariants of the unperturbed particle orbits, this orbit averaged Fokker-Planck equation can symbolically be written as

$$p_0^2 \lambda \sin\theta_0 \frac{\partial f_0(r_0,p_0,\theta_0)}{\partial t} = \frac{\partial}{\partial I_{0i}} p_0^2 \lambda \sin\theta_0 \left(\left\langle \frac{\partial I_{0i}}{\partial \mathbf{u}} \cdot \mathbf{D}^{\mathbf{uu}} \cdot \frac{\partial I_{0j}}{\partial \mathbf{u}} \right\rangle \frac{\partial}{\partial I_{0j}} - \left\langle \frac{\partial I_{0i}}{\partial \mathbf{u}} \cdot \mathbf{F}^{\mathbf{u}} \right\rangle \right) f_0(r_0,p_0,\theta_0) \quad (8)$$

where the brackets $< A > \equiv \tau_B^{-1} \oint (A/|v_\parallel|) ds$ denote the bounce average with the bounce time $\tau_B \equiv \oint (1/|v_\parallel|) ds$, $\lambda = v_{\parallel 0}\tau_B$, and where $\mathbf{u}$ are local momentum coordinates along the particle orbit, and $\mathbf{D}^{\mathbf{uu}}$ and $\mathbf{F}^{\mathbf{u}}$ are local diffusion and convection coefficients, which incorporate collisions, wave driven quasi-linear diffusion and the parallel electric field. Given the strong localization of the wave power deposition, a critical point is the usual treatment of the bounce average as an effective surface average. This can be questioned in any case for trapped particles, but also for passing particles on (low order) rational surfaces. Several authors have treated these issues in the past (see e.g. [30,31]). However, plasma rotation may help to effectively distribute the wave power on a collisional time scale.

The commonly used 'bounce averaged' Fokker-Planck model is obtained when the orbit drifts are neglected and r_0 is treated as a constant along the particle orbits [32]. Many numerical codes exist which solve the bounce averaged quasi-linear Fokker-Planck equation. A review of the latter in the context of ECRH and ECCD has been given in [23]. However, when the finite orbit drifts are retained, the model includes full neoclassical particle dynamics [33]. In particular, one may identify a neoclassical flux within momentum space [34],

$$\Gamma^i_{\mathrm{neo}} = \left\langle \frac{\partial I_{0i}}{\partial \mathbf{u}} \cdot \mathbf{D}^{\mathbf{uu}} \cdot \frac{\partial r_0}{\partial \mathbf{u}} \right\rangle \frac{\partial}{\partial r_0} f_0(r_0,p_0,\theta_0) \qquad i = (\theta_0, p_0) \quad (9)$$

which is responsible for the bootstrap current. The dominant contribution to the bootstrap current has been identified as coming from the singular behaviour of r_0 at the trapped passing boundary [35,36], which introduces a flux across the trapped passing boundary into the passing particle region equal to

$$\Gamma^i_{\mathrm{neo}} \approx \Gamma^{\theta_0}_{\mathrm{neo,t/p}} = \mp\delta(\theta_0 - \theta_{0,\mathrm{t/p}}) \mathbf{D}^{\theta_0\theta_0} w_b \frac{\partial}{\partial r_0} f_0(\mathbf{I}_0), \quad (10)$$

where w_b is the half width of a banana orbit at the trapped passing boundary, and $\mp$ refers to the co- and counter-passing boundaries, respectively. The latter approximation has been used to study bootstrap current in strongly nonthermal plasmas and the possible synergism between bootstrap current and RF current drive [35, 36].

More recently, the synergism of bootstrap current and RF driven current has been revisited using an expansion of the steady state drift kinetic equation in terms of the ratio of the bounce time and the typical time for the radial drift due to the magnetic field gradient and curvature. The leading order then is identical to the steady state bounce averaged equation (i.e. neglecting particle drifts). The first order correction is the sum of the perturbation of the distribution function $\tilde{f}$ due to the finite drifts and the corresponding plasma response g due to collisions and RF fields:

$$\Big\langle D(g) \Big\rangle = -\Big\langle D(\tilde{f}) \Big\rangle \tag{11}$$

where the operator D includes the effects from collisions, quasi-linear RF diffusion, et cetera. Note, that this latter approach formally is identical to the method sketched above in case the radial derivatives occurring in (9) are based on the zero order distribution function only.

As discussed above, the bounce averaged Fokker-Planck model (or expansion of the drift kinetic equation) does not include the effects of anomalous cross field transport. This neglect can only be justified when the collisional slowing down time of relevant particle populations is short compared to their radial diffusion time. In many cases of strong quasi-linear modifications and generation of significant nonthermal electron populations this approximation is not justified. As is well known from detailed comparisons with experiments, anomalous transport can modify significantly both the magnitude of quasi-linear driven nonthermal populations as well as their radial distribution. Consequently, calculations in which radial gradients of the nonthermal distributions play a role, like those of the synergy between bootstrap and RF driven currents, must be taken with some care.

3.2 Self-consistent current density evolution

A common usage of the bounce averaged Fokker-Planck model is calculation of the RF generated noninductive plasma current. However, as a model for the current density evolution the bounce averaged Fokker-Planck model is in itself incomplete. Any change in the current density introduced by the RF waves, is accompanied by a change in the parallel electric field which will tend to counteract the RF driven current. Moreover, the nonthermal populations generated by the RF quasi-linear diffusion may reduce the plasma resistivity significantly. This synergism between RF current drive and the inductive current can be particularly important in cases of counter-current drive, where the decrease in resistivity may offset completely the counter driven RF current (see for example [37]). Consequently, the evolution of the parallel electric field cannot be obtained from a modified Ohm's law, but must be solved self-consistently. The completion of the plasma model requires the introduction of Maxwell's equations for the evolution of the electromagnetic fields. In particular, the loop voltage at a given flux surface may be obtained from Faraday's law:

$$V_{\text{loop}} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{S}, \tag{12}$$

where the rate of change of the magnetic flux through toroidal hole in the flux surface (the right hand side of the equation) contains both the externally generated field from the primary circuit, as well as the contributions from the changes in the plasma current itself. Using the low frequency approximation of Ampére's equation (i.e. neglecting the displacement current) and a simple cylindrical approximation for the poloidal field inside the plasma, one obtains

$$V_{\text{loop}}(r,t) = V_{\text{ext}}(t) - \frac{\mu_0 R h_i}{2} \frac{\partial I_p}{\partial t} - \mu_0 R \int_r^a \frac{\partial I(r')}{r' \partial t} dr', \tag{13}$$

where the terms on the right hand side represent the single loop voltage over the primary winding, V_{ext}, the rate of change of the flux between plasma boundary and the primary winding (with h_i being a geometric constant), and the rate of change of the flux between the plasma boundary and and the internal radius r, respectively. $I(r)$ is the total current enclosed within the radius r, and $I_p = I(a)$ is the total plasma current. The significance of this self-consistent electric field evolution remains to be assessed by the numerical codes.

4 Nonlinear aspects

An extensive review of the nonlinear theory of wave propagation and absorption in the electron cyclotron range of frequencies is given in [38, 39]. These works cover both the nonlinear aspects of wave absorption and the consequences of nonlinear dispersion for wave propagation (self-focusing) and stability of the wave beam (parametric instabilities). They were motivated mostly by the application of very high power pulsed microwave sources (10^2–10^4 MW peak power free electron lasers) for ECRH/ECCD. Wave absorption was discussed mostly in the strongly nonlinear regime. However, nonlinear effects already appear at much lower power. Below, two such effects are discussed. The first concerns nonlinear wave absorption. As shown below, the limits of quasi-linear theory can already be passed at the 1 MW level of currently available gyrotrons. The second concerns the phenomenon of electromagnetically induced transparency (EIT), in which the dispersion of a low power 'signal wave' is changed in the presence of a high power 'pump wave'.

4.1 Nonlinear absorption

The basic assumption of quasi-linear theory breaks down, when the acceleration by the wave of an electron leads to loss of synchronism between the wave phase and gyroperiod during the coherent interaction of the electron with the wave. The nonlinear interaction results in trapping of resonant electrons in a wave period on a time scale given by the nonlinear 'trapping time' τ_{tr}. The importance of nonlinear wave particle interaction can thus be measured in terms of the ratio of a time scale for the coherent interaction between waves and particles, $\tau_{\rm co}$, over the trapping time $\tau_{\rm tr}$:

$$\varepsilon_{NL} \equiv \frac{\tau_{\rm co}}{\tau_{\rm tr}}, \tag{14}$$

where $\varepsilon_{NL} \ll 1$ must hold for wave particle interaction to be (quasi-)linear. With very few exceptions, subsequent passages of an electron through a wave beam in the EC range of frequencies can be treated as incoherent, and $\tau_{\rm co}$ is determined by a single transit through the wave beam. One usually identifies the coherence time with the time it takes the particle to cross the wave beam, i.e.

$$\tau_{\rm co}(w) = \frac{2w}{v_\parallel}. \tag{15}$$

However, this neglects possible inhomogeneities in the wave beam and in the plasma equilibrium parameters along the particle trajectory. The coherence time resulting from linear variations of wave and equilibrium parameters only is [40]

$$\tau_{\rm co}(\text{inhom.}) = \left| v_\parallel \frac{\partial \left(\omega_{ce} - k_\parallel v_\parallel\right)}{\partial s} \right|^{-\frac{1}{2}}, \tag{16}$$

where the derivative is to be taken along the particle trajectory. For a Gaussian beam with a finite radius of curvature this gives (neglecting a possible inhomogeneity of the magnetic field)

$$\tau_{\rm co}(R_c) = \sqrt{\frac{R_c c}{\omega v_\parallel^2}}. \tag{17}$$

The actual coherence time will depend on the combination of all these effects and might be estimated by the minimum of the coherence times given above.

In the case of electron cyclotron waves and for nearly perpendicular propagation, the wave-particle interaction changes mostly the perpendicular energy of the resonant electrons. The relative change between the gyrophase of resonant electrons and the wave phase can be calculated from the rate of change of the relativistic factor $\gamma \equiv \sqrt{1 + p^2/m_e^2c^2}$ and the consequent change of the particle's gyrofrequency

$$\Delta\text{phase}(\tau) = -\omega_{ce} \int_0^\tau dt \int_0^t dt' \partial\gamma/\partial t'. \tag{18}$$

An estimate of the trapping time $\tau_{\rm tr}$ is then obtained by estimating $\partial\gamma/\partial t$ by its quasi-linear approximation and letting $|\Delta\text{phase}(\tau_{\rm tr})| \equiv 2\pi$:

$$\tau_{\rm tr} = \sqrt{\frac{4\pi}{(\partial\gamma/\partial t)_{QL}}}. \tag{19}$$

For a quasi-perpendicularly propagating wave (i.e. $N_\parallel^2 \ll 1$), the quasi-linear rate of change of γ is

$$\frac{\partial m_e c^2 \gamma}{\partial t} \approx v_\perp \left(\frac{1}{\sqrt{2}} \left(eE_- J_{n-1} + eE_+ J_{n+1} \right) + \frac{v_\parallel n \omega_{ce}}{v_\perp \gamma \omega} eE_\parallel J_n \right), \qquad (20)$$

where $E_{-,+,\parallel}$ are the different contributions to the wave electric field: the perpendicular, right and left handed circularly polarized and parallel components, respectively.

Assuming a dominant right-handed or parallel polarization for an X- or O-mode wave beam, respectively, and using the coherence time $\tau_{co}(w)$ coming from the finite beam width, one obtains the following estimates for the measure of nonlinearity expressed in terms of the wave power P_0 in MW, the beam width w, and the wavelength λ

$$\varepsilon_{\rm NL}(w) \approx \begin{cases} 0.4 \quad \tan\theta \ \sqrt{\dfrac{w}{\lambda}}\, P_0^{1/4}[MW] & \text{for } n = 2 \text{ X-mode} \\ \\ 0.4 \sqrt{\tan\theta} \ \sqrt{\dfrac{w}{\lambda}}\, P_0^{1/4}[MW] & \text{for } n = 1 \text{ O-mode,} \end{cases} \qquad (21)$$

where $\theta \equiv \arctan v_\perp / v_\parallel$ is the local pitch angle. For the other harmonics the nonlinearity is multiplied by a factor of the order of $(v/c)^{(n-2)/2,(n-1)/2}$ for X- and O-mode, respectively. With present day beam powers reaching 1 MW, it becomes clear that for both the fundamental O-mode and second harmonic X-mode a considerable part of phase space may well be in the nonlinear regime. In particular a cone in velocity space with sufficiently small $v_\parallel / v_\perp$ will always be in the nonlinear regime. In case of the second harmonic X-mode at perpendicular propagation this coincides with that part of velocity space where most energy is absorbed. In contrast, only a small fraction of the power is absorbed in this region in case of the fundamental harmonic O-mode (see Eq. (20)).

Nonlinearity will affect the power absorption, in particular its effect on and distribution in velocity space. This problem has mostly been treated in the limit of strong nonlinearity $\varepsilon_{NL}(w) \gg 1$, which means that the nonlinear trapping time is much smaller than the transit time. In that case, the existence of an adiabatic invariant and the assumption of wave-particle phase randomization during the beam transit leads to the prediction that resonant particles either retain their energy or make a jump in energy which is equal to the width of the separatrix at the moment the particle is trapped. When the magnetic field is inhomogeneous across the wave beam this picture is altered, as trapped particles are carried to higher (lower) energies as they pass in the direction of increasing (decreasing) magnetic field [40]. At the currently available powers and beam widths, however, the adiabatic limit applies only to a vanishingly small part of phase space with pitch angles of almost exactly 90^o.

In practical cases, a large fraction of phase space may have ε_{NL} of order unity, and neither adiabatic nor quasi-linear theory applies. A recent study of this regime has been undertaken in Ref. [41]. A stochastic mapping technique is applied, in which the transition probabilities for a single beam crossing are computed (subsequent crossings are assumed to be incoherent) and the evolution between crossings is given by the drift kinetic equation. Monte Carlo methods are used to solve the resulting equations and calculate the nonlinear absorption. However, two important approximations have been made in the calculations to obtain the transition probability distributions in [41]: firstly, the wave beam is assumed to be Gaussian with vanishing phase front curvature (i.e. the particle is assumed to pass the waves near the beam waist); secondly, within the wave beam the plasma is approximated as being homogeneous. As noted above, both finite phase front curvature and plasma inhomogeneity may limit the coherence time. Inclusion of these effects into the theoretical model of Ref. [41] and corresponding numerical codes is straightforward. First calculations show that the transition probability distribution changes significantly when $\sqrt{R_c c/\omega} \leq 3w$ [42], which corresponds to the coherence time from the finite beam width (15) and that from a finite phase front curvature (17) becoming of similar magnitude.

An important approximation in many of these studies (in particular [41]) and repeated in the discussion above, is the neglect of the parallel acceleration by the waves. Although reasonable in case of nearly perpendicular wave propagation, this approximation is questionable in cases of oblique propagation as are typical for applications of ECCD. At least in case of second harmonic X-mode, the importance of the parallel acceleration is exemplified by results of quasi-linear modeling, which indicate much stronger nonthermal electron generation at similar local power densities in case of oblique propagation as compared to nearly perpendicular propagation. This is most likely related to the fact that, with oblique propagation, the direction of the quasi-linear diffusion lines up better with the resonance curve in the region of largest power deposition. The inclusion of parallel acceleration significantly complicates the analysis and increases the dimensionality of the problem.

Also, the final remarks of Section 2.2 concerning the evolution of the beam profile apply equally in case of nonlinear absorption from the wave beam (in [41] a Gaussian beam profile is assumed). Even more so, because the nonlinearity, and consequently the nonlinear absorption, will vary over the beam cross-section: the nonlinearity will be largest for the field line crossing the centre of the beam, while ε_{NL} will be small for field lines only touching the edges of the beam. Also the assumption that the energy absorption along a field line is distributed proportional to the wave power will break down in the strongly nonlinear regime and the consequences for the beam profile evolution need to be addressed. Even a nonlocal redistribution of beam energy may occur [43]. Finally, for those cases in which the wave dispersion and polarization depend critically on the plasma response, the nonlinear plasma response will also affect the wave dispersion and polarization: the consequences of the nonlinear plasma response for the warm plasma effects on wave propagation mentioned in Section 2.1 need to be investigated.

4.2 Electromagnetically induced transparency

Another nonlinear effect that may manifest itself at the power levels from currently available gyrotrons is the classical analog of the quantum mechanical phenomenon of electromagnetically induced transparency (EIT) [44]. In the classical case, EIT in plasmas involves three plasma waves: a high power pump wave ($\omega_{\rm pump}, k_{\rm pump}$), a low power probe wave ($\omega_{\rm probe}, k_{\rm probe}$), and a plasma wave that is driven by the beating ($\omega_{\rm beat} = \omega_{\rm probe} - \omega_{\rm pump}, k_{\rm beat} = k_{\rm probe} - k_{\rm pump}$) of the pump and probe waves. The plasma response at the probe frequency and linear in the amplitude of the probe then exists of the usual linear response to the probe wave and a contribution from the nonlinear interaction of the beat plasma wave with the pump wave, i.e. [45]

$$\tilde{j}_{\rm probe} = -e\left(n_e v_{\rm probe} + \frac{1}{2}\tilde{n} v_{\rm pump}\right), \qquad (22)$$

where $v_{\rm pump,probe}$ are the oscillatory velocities driven by the pump and probe waves, respectively, and $\tilde{n}$ is the density perturbation driven by the beating of the pump and probe waves. Under the proper conditions, the nonlinear plasma response from the interaction of the pump and the plasma wave may cancel the resonant plasma response from the probe wave. This opens a window of transparency in the absorption line, characterized by a refractive index close to unity and a very low group velocity as is typical for EIT [45]. First theoretical studies have addressed the phenomenon of classical EIT in plasmas for parallel propagating waves with right handed polarization in the cold plasma limit [45,46]. Subsequently, this work has been extended to take into account warm plasma effects both in a hydrodynamic limit and in a fully kinetic description [47]. A window of transparency is created around the electron cyclotron frequency with a width $\Delta\omega = |\omega_{\rm probe} - \omega_{ce}|$ which is approximately [47, 48]

$$\Delta\omega \approx \max\left\{\sqrt{\omega_{ce}\omega_{pe}\xi_{EC}}, \frac{k_{\rm beat}^2 v_t^2}{\omega_{pe}}\right\}, \qquad (23)$$

where v_t is the thermal velocity and xi_{EC} is a dimensionless nonlinearity parameter, which equals the ratio of the squares of the oscillatory velocity induced by the pump wave and its phase velocity: $\xi_{EC} = |v_{\rm pump}k_{\rm pump}/2\omega_{\rm pump}|^2$.

Probably of more interest for applications in fusion devices is the case of perpendicular propagation. A case of induced absorption rather than transparency was found for a perpendicularly propagating X-mode probe wave, where the nonlinear interaction of the pump and a plasma wave results in an increase of the right handed polarized component at the fundamental cyclotron resonance and a concomitant increase in the resonant absorption [49]. However, for a perpendicularly propagating X-mode wave the possibility of EIT has been shown in the evanescent region between the slow and fast X-mode branches, i.e. between the Upper-Hybrid resonance ω_{UH} and the low density cut-off ω_+ [50]. The efficient excitation of plasma waves in this case requires an obliquely propagating pump wave with the conditions

$$\begin{aligned} |\omega_{\rm beat} - \omega_{pe}| &\ll \omega_{\rm beat}, \omega_{pe}, \\ |k_{b\perp}| &\ll |k_{b\parallel}| = |k_{p\parallel}|. \end{aligned} \qquad (24)$$

In the same paper, wave dispersion under the conditions of EIT in an inhomogeneous plasma has been analyzed. Efficient tunneling from the EIT induced branch to the slow X-mode branch is found to provide a route for radiation from the high density plasma interior to escape through the evanescent region to the vacuum [50]. Thus far the latter conclusions are based on an analysis based on the cold plasma response only. Important challenges remain. A first challenge is the extension of the analysis to the warm plasma case within a kinetic model. This should also bring the electron-Bernstein waves into the picture. The final challenge will be to create a scenario for practical application of EIT, for example, as a diagnostics: allowing radiation generated in the plasma interior to escape through the evanescent region to the vacuum where it can be measured.

5 Summary and Outlook

As illustrated in the foregoing sections our theoretical framework for the description of propagation and absorption of electron cyclotron waves is still far from complete. Challenges remain even in linear theory of wave propagation in (in)homogeneous plasma: what are the exact wave energy density and flux, what are the consequences for wave beams (their trajectory, amplitude and phase profiles) of the strong variation in wave dispersion and absorption over the cyclotron resonance zone? These and more questions apply equally to electron-Bernstein waves. What is the importance of relativistic effects for electron-Bernstein waves? It is shown how the corresponding quasi-linear plasma response can be calculated including the effects of finite drift orbit widths. This allows to tackle problems like those on the synergy of RF and bootstrap driven currents. Finally, many challenges remain in nonlinear theory. Nonlinear absorption inescapably plays a role even at the 1 MW power levels of currently available gyrotrons. Also, only modest power levels are predicted to be required to achieve an effect as electromagnetically induced transparency. In particular, this latter development promises to yield some surprisingly new applications. In short, many open questions remain and, keeping an open mind, we may be surprised by some unexpected answers.

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