TO THE THEORY OF CYCLOTRON MASER WITHOUT INVERSION.

(THE CYCLOTRON INSTABILITY IN THE NONRESONANT ELECTRON MEDIUM)

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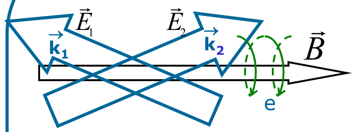
Introduction

This work is devoted to the investigation of novel regimes of cyclotron instability, realized in the form of parametric instability of two waves with close frequencies coupled in the modulated medium of charged particles. It is important that such ensembles of particles are stable against generation of the monochromatic radiation, so that these regimes can be named as ***"inversionless"*** by analogy with quantum effect of Lasing Without Inversion [1,2]. The first effect that was proposed [2] is the generation of cyclotron HF radiation by a modulated ensemble of resonant electrons; in this regime the amplification mechanism corresponds to the parametric interaction with modulated active susceptibility. Recently in papers [3,4] it was predicted that the amplification of bichromatic high-frequency radiation in the presence of low-frequency modulation of electrons is possible even if there is no energy exchange between waves and particles without modulation. Such a medium is ***reactive*** with respect to the monochromatic radiation - due to ***the absence of resonant particles***. In this regime ***not scattering***, but ***simultaneous amplification of two waves*** is realized.

Here we investigate the NONRESONANT REGIME OF INVERSIONLESS MASER

The field

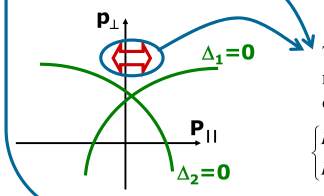


This regime of "nonresonant" instability can be realized for the waves with close frequencies propagating in the directions different from perpendicular with respect to the magnetic field. Optimally: two waves propagate along the magnetic field in opposite directions.

$$\mathbf{E}(z,t) = \mathrm{Re}\,\mathbf{e}_+ \sum_{j=1}^{2} E_j \exp(ik_j z - i\omega_j t) \qquad \omega_1 \approx \omega_2,\ k_1 \approx -k_2$$

The ensemble of particles. The necessary type of modulation.

$$\Delta_j = \omega_j - \frac{eB}{mc\gamma} - k_{\parallel j} V_\parallel$$



The particles are not resonant to the fields. $\Delta_j \neq 0$

$\omega_1 - \omega_2 \approx (k_1 - k_2)V_\parallel$

Parametric synchronism

The active response is zero, $\sigma \propto \int \hat{L} f \delta(\Delta_j) d^3 p = 0$

the reactive response is not zero $\chi \propto \int \hat{L} f / \Delta_j \, d^3 p \neq 0$

The modulation should provide the oscillations of reactive responses in opposite phases - due to oscillation of Δ_1 and Δ_2 in opposite phases.

$$f(p_\parallel, p_\perp) = f_0 + f_M \cos(\varphi_M + (k_1 - k_2)z - (\omega_1 - \omega_2)t)$$

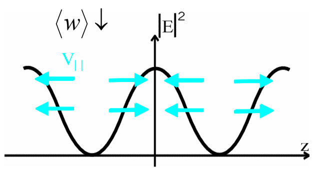
$$f_M(p_\parallel, p_\perp) = -f_M(-p_\parallel, p_\perp)$$

$$\begin{cases} \dot{E}_1 + (\sigma_0 + i\chi_0)E_1 = -(\sigma_M + i\chi_M)e^{i\varphi_M} E_2 \\ \dot{E}_2 + (\sigma_0 + i\chi_0)E_2 = (\sigma_M + i\chi_M)e^{-i\varphi_M} E_1 \end{cases} \rightarrow \text{instability} \quad |\chi_M| > \sigma_0 = 0$$

The mechanism of energy exchange

The explanation in one-frequency frame ($\omega_1=\omega_2$).

(The energy exchange between the field and the medium is absent in usual scattering)



Modulation of $V_\parallel$ on Z → Energy exchange of the field and the ensemble of particles

Change of averaged energy of relativistic electron

$$\frac{d}{dt}\langle w \rangle = -\frac{e^2}{m\gamma_0}\frac{1}{2\omega\Delta_0}\left(1 - \frac{V_\perp^2}{c^2}\frac{\omega}{\Delta_0} + \frac{\omega}{\Delta_0}\left(2\frac{V_\perp^2}{c^2}\frac{\omega}{\Delta_0} - 1\right)\right)\left(V_\parallel \frac{\partial}{\partial Z}|E|^2(Z(t))\right)$$

What is the source of necessary modulation?

1. Preliminary action of longitudinal LF wave
2. Self-action of two HF waves (nonlinear source of modulation)

$$\frac{d}{dt}P_\parallel = \frac{e^2}{m\gamma_0}\frac{1}{2\Delta_0^2}\frac{V_\perp^2}{c^2}\left(\frac{\partial}{\partial Z}|E|^2(Z)\right)$$

Damage of initial modulation → saturation of instability

1. Ballistic or dynamic transformation of initial modulation of $V_\parallel$ to the modulation of density.

$t_{sat} \sim 1/\Delta_M$

$\Delta_M \sim 2k\Delta V_\parallel$ Ballistic (linear) regime

$\Delta_M \sim \dfrac{eEV_\perp \omega}{mc\gamma\Delta_0}$ Dynamic (nonlinear) regime

2. Destroying influence of excited longitudinal plasma field

$t_{sat} \sim 1/\omega_p$

Kinetic theory (linear, quasivacuum approximations, $t \ll t_{sat}$)

$$\begin{cases} \dot{E}_1 = -e^{i\varphi_M}(\Gamma t + iD)E_2, \\ \dot{E}_2 = +e^{-i\varphi_M}(\Gamma t + iD)E_1. \end{cases}$$

Unstable exponential mode $E_j \propto \exp\int_0^t \mu(\tau)d\tau$

$$(E_1/E_2)_+ = -i(D/|D|)\exp(i\varphi_M)$$

$$\mu_+ = |D| - i\Gamma t(D/|D|)$$

increment

Time-dependent shift of frequency $\Delta\omega(t)$- approaching to the resonance

$$\Gamma = -\frac{2\pi e^2 V_\perp^2 \omega}{m^2\gamma^2 c^2}\int dp_\perp dp_\parallel f_M 2kp_\parallel \frac{1}{\Delta_0^2}$$

$$D = \frac{2\pi e^2 V_\perp^2 \omega}{m^2\gamma^2 c^2}\int dp_\perp dp_\parallel f_M 2kp_\parallel \frac{1}{\Delta_0^3}.$$

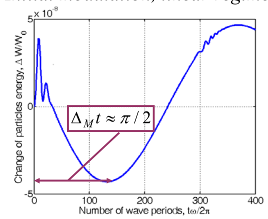
Different scenarios of the amplification process. Efficiency.

1 $\omega_p, \mu \ll \Delta_M$ **(Low density)**

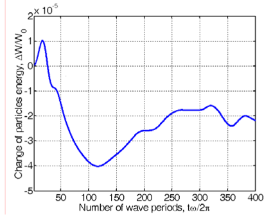
Efficiency $\dfrac{\Delta\langle W\rangle}{W_0} = -\left(\dfrac{e|E|}{mc\omega}\right)^2 \dfrac{V_\perp^2/c^2}{2\gamma_R(\gamma_R - 1)}\left(\dfrac{\omega}{\Delta_0}\right)^3 \gtrsim \dfrac{\Delta_M^2}{\omega\Delta_0} \ll 1$

Gain $\dfrac{\Delta I}{I_0} = \dfrac{2\mu}{\Delta_M} \ll 1$

Initial modulation, linear regime



Nonlinear modulation

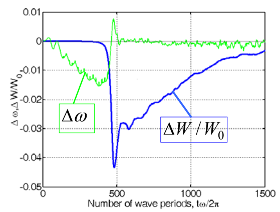


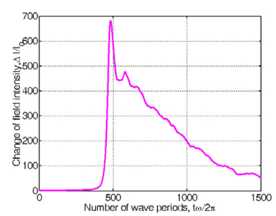
2 $\mu \gg \Delta_M, \omega_p$

Regime of enhanced parametric instability

Efficiency $\dfrac{\Delta\langle W\rangle}{W_0} \sim \dfrac{1}{\gamma}\left(\dfrac{2p_\perp\omega_p}{mc\omega}\right)^{2/3} \gtrsim 5\%$

Gain $\dfrac{\Delta I}{I_0} = \left(\dfrac{\omega_p^4 p_\perp}{\omega^4 mc}\right)^{2/3} \gg 1$





$\lambda = 1mm$

$N = 7 * 10^9 cm^{-3}$

$I_{max} = 500 \dfrac{kW}{cm^2}$

$\dfrac{\langle P \rangle}{V} = 50 \dfrac{kW}{cm^3}$

The parametric "nonresonant" interaction- amplification+frequency shift → $\Delta\omega(t)\to$ to the resonance → Strengthening of the parametric coupling → Increase of energy exchange rate → Nonlinear stage of interaction. Saturation of instability

Conclusion

The demonstrated regime of instability of two waves in "nonresonant" medium of charged particles conflicts with the customary Manley-Rowe relation. This regime is realized in conditions, typical for induced scattering process. These contradictions are canceled by taking into account that demonstrated energy extraction process is limited in time of interaction. On the contrary the induced scattering is asymptotic process, characterized by finished bunching of electrons in definite phase with the beat wave. We have shown that before this bunching process finishes the essential energy exchange process between medium and the field can take place.

The general conclusion can be made:
The regimes of electromagnetic field – medium interaction, agreed with the quantum conservation laws, are settled asymptotically. The time of this transitional process depends on a particular system.

References

[1] Kocharovskaya O.,Phys.Rep.,1992,**219**,175 [2] Gaponov-Grekhov A.V., Tokman M.D. JETP, 1997,**85**,640
[3] Erukhimova M.A., Tokman M.D., JETP,2000,**91**,255 [4] Erukhimova M.A., Tokman M.D., Radiophysics and Quantum Electronics, 2003,**45**,249