PECULIARITIES OF LONGITUDINAL PROPAGATION OF MICROWAVE WITH FREQUENCY NEAR THE ELECTRON CYCLOTRON FREQUENCY IN MAGNETIZED PLASMA

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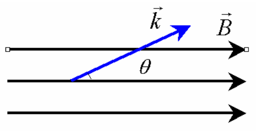


$\omega_B = \dfrac{eB}{m_e c}$ - electron cyclotron frequency, $\omega_L^2 = \dfrac{4\pi N e^2}{m}$ - plasma frequency, $\beta_T = \sqrt{\dfrac{T_e}{mc^2}}$

$\vec{k} = \vec{N}_i \dfrac{\omega}{c}$ $\quad u = \dfrac{\omega_B^2}{\omega^2}$ $\quad v = \dfrac{\omega_L^2}{\omega^2}$ $\quad q = \dfrac{v}{u}$

Dense plasma absorption coefficient [A.I. Akhiezer, I.A. Akhiezer,. Plasma Electrodynamics, Pergamon Press, Oxford. (1975)]:

$$\frac{\omega_L^2}{\omega_B^2 \beta_T N_j \cos\theta} \equiv \sigma >> 1 \qquad \frac{k_\perp^2 \upsilon_T^2}{\omega_B^2} << 1 \qquad \frac{\omega - l\omega_B}{\sqrt{2}\omega N_j \beta_T \cos\theta} >> 1, \text{ for } l \neq 1 \longrightarrow \text{Dispersion equation}$$

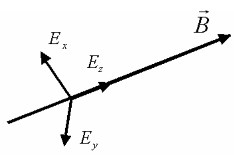
- Zero order ⟶ $\operatorname{Re} N_j$
- First order ⟶ $\operatorname{Im} N_j$

For extraordinary wave: $\theta \to 0$

$$\operatorname{Re} N_j \approx \frac{\sqrt{2(1-q)}}{\theta} \qquad \operatorname{Im} N_j \propto \frac{1}{\theta^4}$$

$\Rightarrow$ ~~$\sigma >> 1$~~ ⟶ **?**

Stix components of electric field and dielectric tensor in the cold plasma approximation [Timofeev A.V. (1992)]:



$$E_\pm = \frac{E_x \pm iE_Y}{\sqrt{2}} \qquad E_\parallel = E_z$$

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_+ & 0 & 0 \\ 0 & \varepsilon_- & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \qquad \varepsilon_+ = 1 - \frac{v}{1+\sqrt{u}} \qquad \varepsilon_- = 1 - \frac{v}{1-\sqrt{u}} \qquad \varepsilon_\parallel = 1 - v$$

Solution of dispersion equation for nearly longitudinal propagation slow waves with frequency near electron cyclotron frequency in cold magnetized plasma.

$\varepsilon_- >> 1$

$\theta << 1$

$N >> 1$

$$N^2 = \frac{2\varepsilon_- \varepsilon_{II}}{2\varepsilon_{II} + \theta^2 \varepsilon_-} + O(\theta^2)$$

Dielectric tensor in the dense plasma For frequency near electron cyclotron frequency

“Warm” plasma.

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_+ & \underline{i\alpha} & 0 \\ \underline{i\alpha} & \varepsilon'_- - \underline{4i\alpha} & \underline{\sqrt{2}\xi} \\ 0 & \underline{\sqrt{2}\xi} & \varepsilon_\parallel + \underline{d} \end{pmatrix}$$

Approximations:

$\alpha \sim \sigma^{-1} << \varepsilon_+ \sim 1 << \varepsilon'_- \sim \sigma$ - Dense plasma

$d \sim \theta^2 << \varepsilon_\parallel \sim 1$ - Nearly longitudinal propagation

$\xi << N_\parallel N_\perp$ - Slow wave

Approximate dispersion relation for nearly longitudinal propagation of slow wave in a dense plasma :

$$N^2 = \frac{2\varepsilon_\parallel \varepsilon'_-(N)}{2\varepsilon_\parallel + \theta^2 \varepsilon'_-(N)}$$

$$\varepsilon'_- = 1 + \frac{i\omega_L^2}{\omega^2 \beta_T N \cos\theta}\sqrt{\frac{\pi}{2}} W(Z) \qquad \alpha = \frac{\beta_T \omega_L^2 N \sin^2\theta}{2\omega_B^2}\sqrt{\frac{\pi}{2}} W(Z) \qquad \xi = \frac{\omega_L^2}{2\omega\omega_B} tg\theta \left(1 + i\sqrt{\pi} W(Z) Z\right)$$

$$d = \frac{\omega_L^2}{2\omega_B^2}\frac{\omega - \omega_B}{\omega} tg^2\theta \left(1 + i\sqrt{\pi} W(Z) Z\right) \qquad W(Z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^Z e^{\xi^2} d\xi\right) \qquad Z = \frac{\omega - \omega_B}{\sqrt{2} N \omega \beta_T \cos\theta}$$

Analytical solution approximate dispersion equation:

Center of absorption line: $\frac{\omega - \omega_B}{\sqrt{2} N \omega \beta_T \cos\theta} \leq 1$

$$\frac{\varepsilon'_- \theta^2}{2\varepsilon_\parallel} >> 1 \to N^2 = \frac{2\varepsilon_\parallel}{\theta^2} - \frac{4\varepsilon_\parallel^2}{\theta^4 \varepsilon'_-}$$

$$\mathrm{Im}\, N = \beta_T \sqrt{\frac{2}{\pi}} \frac{2\varepsilon_\parallel^2 \omega^2}{\theta^4 \omega_L^2} \frac{\mathrm{Re}(W(Z))}{|W(Z)|^2}\bigg|_{Z=Z(\mathrm{Re}\, N)}$$

$$\mathrm{Re}\, N = \frac{\sqrt{2\varepsilon_\parallel}}{\theta}$$

$$\frac{\omega_L^2 \theta^3}{\varepsilon_\parallel^{3/2} \beta_T \omega_B^2} >> 1$$

$$\frac{\theta^2 \varepsilon'_-}{2\varepsilon_\parallel} << 1 \to N^2 = \tilde{\varepsilon}'_- - \frac{\theta^2 \tilde{\varepsilon}'^2_-}{2\varepsilon_\parallel} + 1 \qquad \tilde{\varepsilon}_- = i \frac{\omega_L^2}{\omega^2 N \beta_T} \sqrt{\frac{\pi}{2}} W(Z)$$

$N = N_0 + \delta N$ N_0 - solution of dispersion equation for strictly longitudinal propagation

[Ginzburg V.L. (1967)]

$$\delta N \approx \left(\frac{1}{N_0} - \frac{\theta^2}{2\varepsilon_\parallel} i \frac{\omega_L^2}{\omega^2 \beta_T} \sqrt{\frac{\pi}{2}} W(Z_0)\right)\left(3 - 2\left(Z_0^2 - \frac{i}{\sqrt{\pi}} \frac{Z_0}{W(Z_0)}\right)\right)^{-1} \qquad Z_0 = \frac{\omega - \omega_b}{\sqrt{2} N_0 \beta_T \omega}$$

$$\frac{\theta^2}{\varepsilon_\parallel}\left(\frac{\omega_L^2}{\omega_B^2 \beta_T}\right)^{2/3} << 1$$

Wing of absorption line: $Z >> 1; \mathrm{Im}\, Z << \mathrm{Re}\, Z$

$$\varepsilon'_- \approx 1 - \frac{\omega_L^2}{\omega(\omega - \omega_B)} + i\frac{\delta}{N}; \delta = \frac{\omega_L^2}{\omega^2 \beta_T} e^{-Z^2} \to N^2 \approx \frac{2\varepsilon_\parallel \varepsilon_-}{2\varepsilon_\parallel + \theta^2 \varepsilon_-} + i \frac{2\varepsilon_\parallel \delta}{(2\varepsilon_\parallel + \theta^2 \varepsilon_-) N} - i \frac{2\varepsilon_\parallel \theta^2 \varepsilon_- \delta}{(2\varepsilon_\parallel + \theta^2 \varepsilon_-)^2 N} \to \mathrm{Re}\, N = \sqrt{\frac{2\varepsilon_\parallel \varepsilon_-}{2\varepsilon_\parallel + \theta^2 \varepsilon_-}}; \mathrm{Im}\, N = \frac{\varepsilon_\parallel \delta}{\varepsilon_-(2\varepsilon_\parallel + \theta^2 \varepsilon_-)}$$

Numerical solution approximate dispersion equation:

$$n = N \beta^{\frac{1}{3}} q^{-\frac{1}{3}}$$

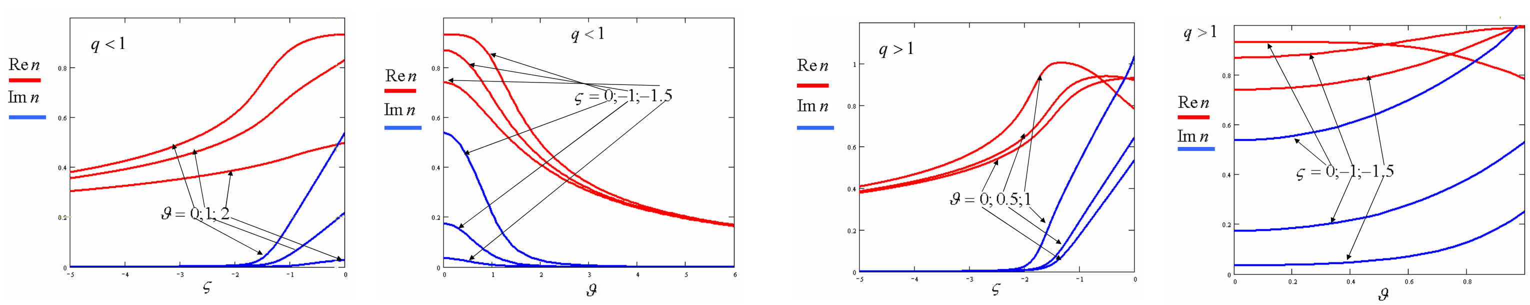
$$\xi = \frac{\omega - \omega_B}{\sqrt{2} \omega \beta_T^{\frac{2}{3}} q^{\frac{1}{3}}}$$

$$\vartheta^2 = \theta^2 \frac{q^{\frac{2}{3}}}{\beta^{\frac{2}{3}} |1 - q|}$$

$$\to n^2 = \frac{\frac{i}{n}\sqrt{\frac{\pi}{2}} W\left(\frac{\xi}{n}\right)}{1 \pm i \frac{\vartheta^2}{n} \sqrt{\frac{\pi}{2}} W\left(\frac{\xi}{n}\right)} \to n^3 = F(n, \xi, \theta) = i\sqrt{\frac{\pi}{2}} W\left(\frac{\xi}{n}\right)\left(1 \mp \vartheta^2 n^2\right)$$

Upper sign for $q < 1$

Lower sing for $q > 1$



Conclusions.

✓ Approximate dispersion relation for nearly longitudinal propagation of slow wave in a dense plasma with finite electron temperature has been derived.

✓ Analytical (perturbation technique), numerical and combined investigations of the dispersion relation solutions have been performed and correspondence to known limiting cases have been checked.

✓ The results obtained are of importance for the modeling of microwave power deposition into ECR discharge of axisymmetric mirror magnetic trap with the longitudinal launch of rf power.