

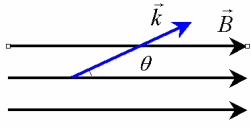
PECULIARITIES OF LONGITUDINAL PROPAGATION OF MICROWAVE WITH FREQUENCY NEAR THE ELECTRON CYCLOTRON FREQUENCY IN MAGNETIZED PLASMA



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$$\omega_B = \frac{eB}{m_e c} \quad \text{electron cyclotron frequency.} \quad \omega_L^2 = \frac{4\pi N e^2}{m} \quad \text{plasma frequency.} \quad \beta_T = \sqrt{\frac{T_e}{m c^2}}$$

$$\vec{k} = \vec{N}_i \frac{\omega}{c} \quad u = \frac{\omega_B^2}{\omega^2} \quad v = \frac{\omega_L^2}{\omega^2} \quad q = \frac{v}{u}$$

Dense plasma absorption coefficient [A.I. Akhiezer, I.A. Akhiezer, Plasma Electrodynamics, Pergamon Press, Oxford. (1975)]:

$$\frac{\omega_L^2}{\omega_B^2 \beta_T N_j \cos \theta} \equiv \sigma \gg 1 \quad \frac{k_{\perp}^2 v_T^2}{\omega_B^2} \ll 1 \quad \frac{\omega - l\omega_B}{\sqrt{2}\omega N_j \beta_T \cos \theta} \gg 1, \text{ for } l \neq 1 \rightarrow \text{Dispersion equation} \begin{cases} \text{Zero order} \rightarrow \text{Re } N_j \\ \text{First order} \rightarrow \text{Im } N_j \end{cases}$$

For extraordinary wave: $\theta \rightarrow 0$

$$\begin{aligned} \text{Re } N_j &\approx \frac{\sqrt{2(1-q)}}{\theta} \\ \text{Im } N_j &\propto \frac{1}{\theta^4} \end{aligned} \rightarrow \sigma \not> 1 \rightarrow ?$$

Six components of electric field and dielectric tensor in the cold plasma approximation [Timofeev A.V. (1992)]:

$$E_{\pm} = \frac{E_x \pm iE_y}{\sqrt{2}} \quad E_{\parallel} = E_z \Rightarrow \epsilon_{ij} = \begin{pmatrix} \epsilon_+ & 0 & 0 \\ 0 & \epsilon_- & 0 \\ 0 & 0 & \epsilon_{\parallel} \end{pmatrix} \quad \begin{aligned} \epsilon_+ &= 1 - \frac{v}{1+\sqrt{u}} & \epsilon_- &= 1 - \frac{v}{1-\sqrt{u}} \\ \epsilon_{\parallel} &= 1 - v \end{aligned}$$

Solution of dispersion equation for nearly longitudinal propagation slow waves with frequency near electron cyclotron frequency in cold magnetized plasma.

$$\begin{aligned} \epsilon_- \gg 1 \\ \theta \ll 1 \\ N \gg 1 \end{aligned} \Rightarrow N^2 = \frac{2\epsilon_- \epsilon_{\parallel}}{2\epsilon_{\parallel} + \theta^2 \epsilon_-} + O(\theta^2)$$

Dielectric tensor in the dense plasma For frequency near electron cyclotron frequency

“Warm” plasma.

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_+ & i\alpha & 0 \\ i\alpha & \epsilon'_- - 4i\alpha & \sqrt{2}\xi \\ 0 & \sqrt{2}\xi & \epsilon_{\parallel} + d \end{pmatrix}$$

Approximations:

- $\alpha \sim \sigma^{-1} \ll \epsilon_+ \sim 1 \ll \epsilon'_- \sim \sigma$ - Dense plasma
- $d \sim \theta^2 \ll \epsilon_{\parallel} \sim 1$ - Nearly longitudinal propagation
- $\xi \ll N_{\parallel} N_{\perp}$ - Slow wave

Approximate dispersion relation for nearly longitudinal propagation of slow wave in a dense plasma :

$$N^2 = \frac{2\epsilon_{\parallel} \epsilon'_-(N)}{2\epsilon_{\parallel} + \theta^2 \epsilon'_-(N)}$$

$$\begin{aligned} \epsilon'_- &= 1 + \frac{i\omega_L^2}{\omega^2 \beta_T N \cos \theta} \sqrt{\frac{\pi}{2}} W(Z) & \alpha &= \frac{\beta_T \omega_L^2 N \sin^2 \theta}{2\omega_B^2} \sqrt{\frac{\pi}{2}} W(Z) & \xi &= \frac{\omega_L^2}{2\omega\omega_B} \text{tg} \theta (1 + i\sqrt{\pi} W(Z) Z) \\ d &= \frac{\omega_L^2}{2\omega_B^2} \frac{\omega - \omega_B}{\omega} \text{tg}^2 \theta (1 + i\sqrt{\pi} W(Z) Z) & W(Z) &= e^{-Z^2} \left(1 + \frac{2i}{\sqrt{\pi}} \int_0^Z e^{\xi^2} d\xi \right) & Z &= \frac{\omega - \omega_B}{\sqrt{2} N \omega \beta_T \cos \theta} \end{aligned}$$

Analytical solution approximate dispersion equation:

Center of absorption line: $\frac{\omega - \omega_B}{\sqrt{2N\omega\beta_T} \cos \theta} \leq 1$

$$\frac{\varepsilon'_- \theta^2}{2\varepsilon_{||}} \gg 1 \rightarrow N^2 = \frac{2\varepsilon_{||}}{\theta^2} - \frac{4\varepsilon_{||}^2}{\theta^4 \varepsilon'_-} \quad \left. \begin{array}{l} \text{Im } N = \beta_T \sqrt{\frac{2}{\pi} \frac{2\varepsilon_{||}^2 \omega^2 \text{Re}(W(Z))}{\theta^4 \omega_L^2 |W(Z)|^2}} \\ \text{Re } N = \frac{\sqrt{2\varepsilon_{||}}}{\theta} \end{array} \right\}_{Z=Z(\text{Re } N)}$$

$$\frac{\omega_L^2 \theta^3}{\varepsilon_{||}^{3/2} \beta_T \omega_B^2} \gg 1$$

$$\frac{\theta^2 \varepsilon'_-}{2\varepsilon_{||}} \ll 1 \rightarrow N^2 = \tilde{\varepsilon}'_- - \frac{\theta^2 \tilde{\varepsilon}'_-{}^2}{2\varepsilon_{||}} + 1 \quad \tilde{\varepsilon}'_- = i \frac{\omega_L^2}{\omega^2 N \beta_T} \sqrt{\frac{\pi}{2}} W(Z)$$

$N = N_0 + \delta N$ N_0 - solution of dispersion equation for strictly longitudinal propagation

[Ginzburg V.L. (1967)]

$$\delta N \approx \left(\frac{1}{N_0} - \frac{\theta^2}{2\varepsilon_{||}} i \frac{\omega_L^2}{\omega^2 \beta_T} \sqrt{\frac{\pi}{2}} W(Z_0) \right) \left(3 - 2 \left(Z_0^2 - \frac{i}{\sqrt{\pi}} \frac{Z_0}{W(Z_0)} \right) \right)^{-1} \quad Z_0 = \frac{\omega - \omega_b}{\sqrt{2N_0} \beta_T \omega}$$

$$\frac{\theta^2}{\varepsilon_{||}} \left(\frac{\omega_L^2}{\omega_b^2 \beta_T} \right)^{2/3} \ll 1$$

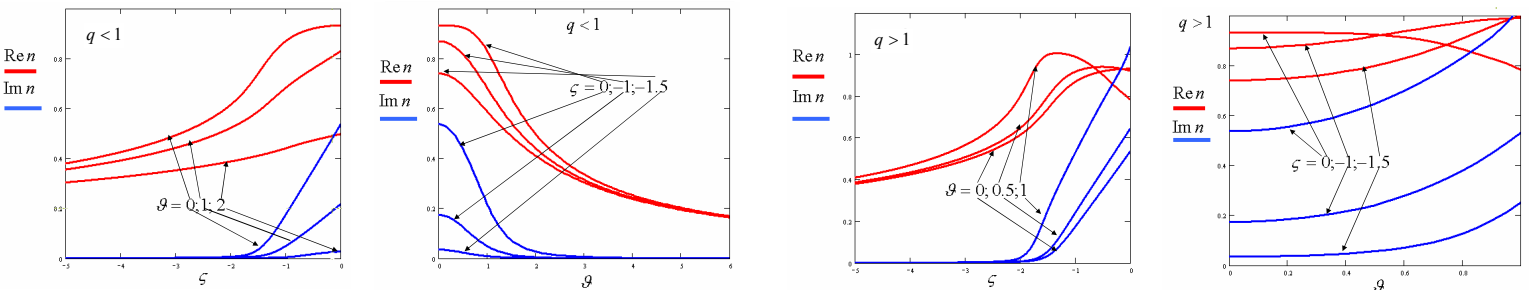
Wing of absorption line: $Z \gg 1; \text{Im } Z \ll \text{Re } Z$

$$\varepsilon'_- \approx 1 - \frac{\omega_L^2}{\omega(\omega - \omega_B)} + i \frac{\delta}{N}; \delta = \frac{\omega_L^2}{\omega^2 \beta_T} e^{-Z^2} \rightarrow N^2 \approx \frac{2\varepsilon_{||} \varepsilon_-}{2\varepsilon_{||} + \theta^2 \varepsilon_-} + i \frac{2\varepsilon_{||} \delta}{(2\varepsilon_{||} + \theta^2 \varepsilon_-) N} - i \frac{2\varepsilon_{||} \theta^2 \varepsilon_- \delta}{(2\varepsilon_{||} + \theta^2 \varepsilon_-)^2 N} \rightarrow \text{Re } N = \sqrt{\frac{2\varepsilon_{||} \varepsilon_-}{2\varepsilon_{||} + \theta^2 \varepsilon_-}}; \text{Im } N = \frac{\varepsilon_{||} \delta}{\varepsilon_- (2\varepsilon_{||} + \theta^2 \varepsilon_-)}$$

Numerical solution approximate dispersion equation:

$$\left. \begin{array}{l} n = N \beta_T^{1/3} q^{1/3} \\ \xi = \frac{\omega - \omega_B}{\sqrt{2\omega\beta_T} q^{1/3}} \\ g^2 = \theta^2 \frac{q^2}{\beta_T^{2/3} |1-q|} \end{array} \right\} \rightarrow n^2 = \frac{i}{n} \sqrt{\frac{\pi}{2}} W\left(\frac{\xi}{n}\right) \rightarrow n^3 = F(n, \xi, \theta) = i \sqrt{\frac{\pi}{2}} W\left(\frac{\xi}{n}\right) (1 \mp g^2 n^2)$$

Upper sign for $q < 1$
Lower sign for $q > 1$



Conclusions.

- ✓ Approximate dispersion relation for nearly longitudinal propagation of slow wave in a dense plasma with finite electron temperature has been derived.
- ✓ Analytical (perturbation technique), numerical and combined investigations of the dispersion relation solutions have been performed and correspondence to known limiting cases have been checked.
- ✓ The results obtained are of importance for the modeling of microwave power deposition into ECR discharge of axisymmetric mirror magnetic trap with the longitudinal launch of rf power.