THEORETICAL STUDY OF UNDULATOR INDUCED TRANSPARENCY IN MAGNETOACTIVE PLASMA

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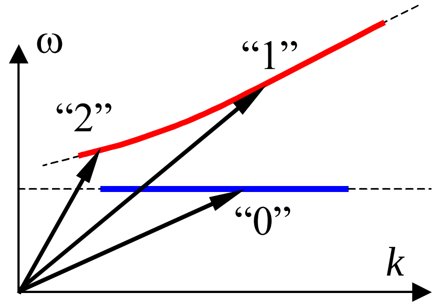
Introduction: EIT in quantum and classical systems

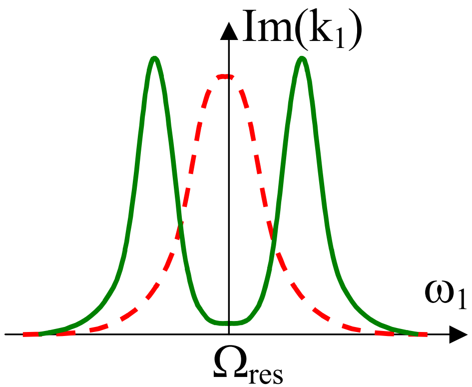
- Variant of parametric 3-wave interaction



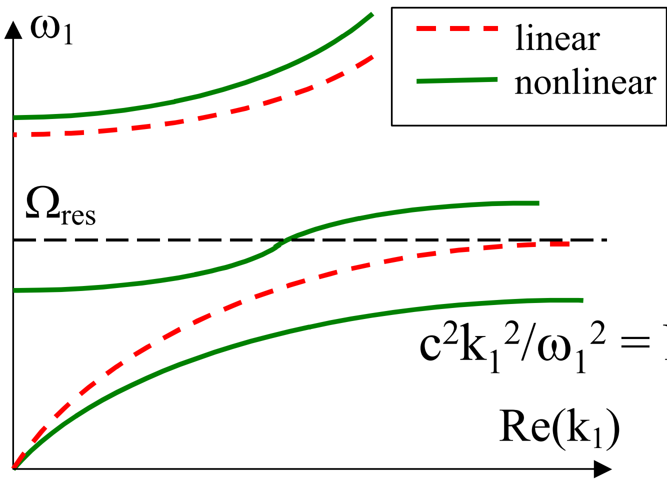
$$\omega_1=\omega_2+\omega_0$$
$$k_1=k_2+k_0$$

- High-power pumping wave «2» «controls» the propagation of probe wave «1»

$|E_2| >> E_{eff}$



- Significant modification of probe wave dispersion

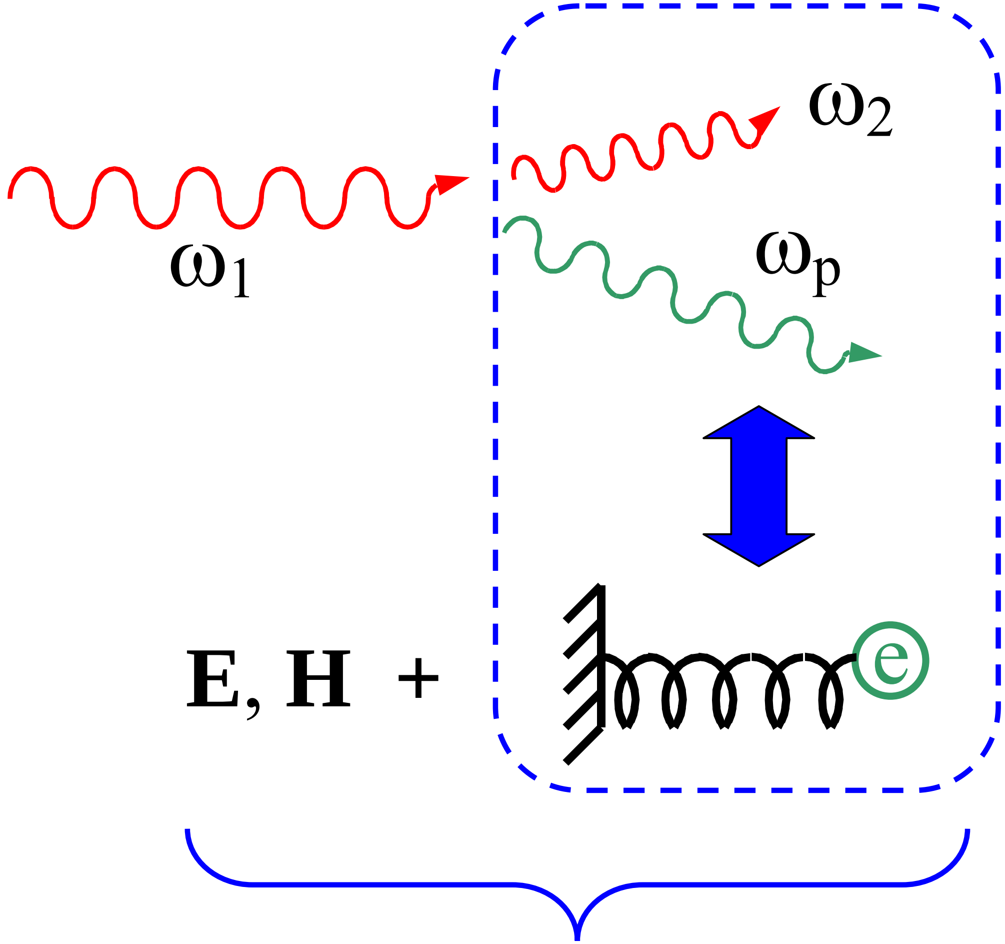


$$c^2k_1{}^2/\omega_1{}^2 = N^2_{eff}(\omega_1, |E_2|^2)$$

- Common effect for quantum (3-level Λ-scheme) and classical (magnetoactive plasma) systems

Litvak A. G., Tokman M. D. // Phys. Rev. Lett. 2002.
Kryachko A. Yu., Litvak A. G., Tokman M. D. // JETP. 2002, Nucl. Fusion. 2004

Interpretation of EIT effect



Significant modification of photon (ω_1, k_1) in medium

Modification of propagation regime for probe wave

photon (ω_1, k_1) in medium

- EIT in plasma: excitation of collective degrees of freedom (plasma oscillations: $\omega_0=\omega_1-\omega_2\rightarrow\omega_p$)

$V_{gr}/c \ll 1 \Rightarrow$ spatial compression $\Rightarrow$

compression of incident wave power in plasma oscillations $\Rightarrow$

possible scheme of accelerator

$$\frac{\text{Plasma oscillation energy}}{\text{Total energy of EIT medium}} = 1-\frac{\omega_2}{\omega_1}$$

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most effective if $\omega_2=0 \Rightarrow$ **magnetic undulator**

(undulator induced reansparency - UIT)

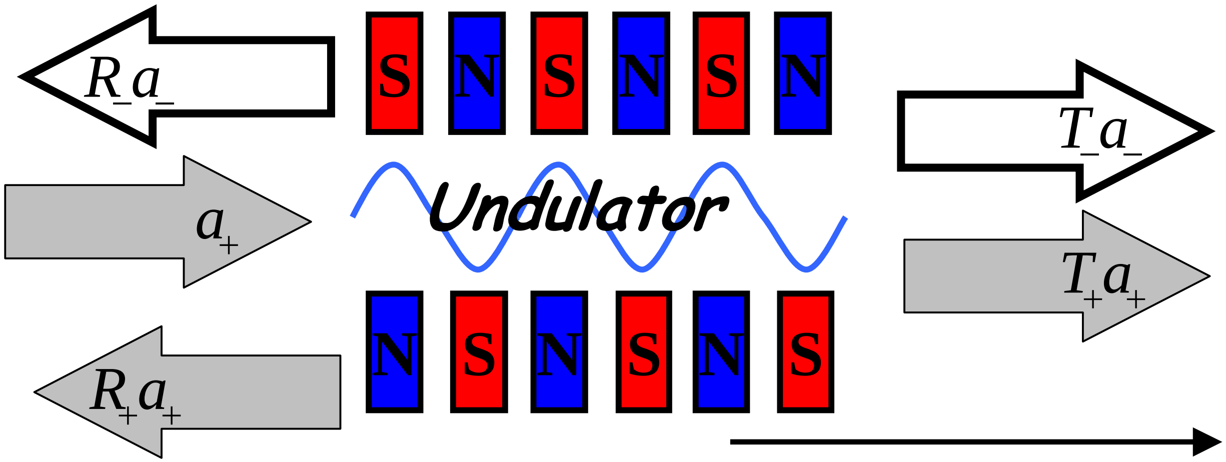
External/undulator magnetic field	10 kG/0.5 kG
Undulator period	1 cm
Incident radiation intencity	1 kW/cm^2
Plasma oscillations phase velocity	0.3 c
Accelerated particles energy	20 MeV

} **Ion acceleration**

Shvets G., Wurtele J. S. // Phys. Rev. Lett., 2002, **89**, 115003
Hur M. S., Wurtele J. S., Shvets G. // Phys. of Plasmas, 2003, **10**, 3004

Formulation of the problem

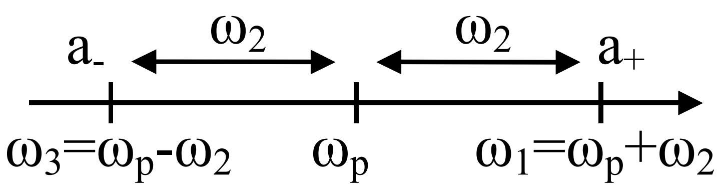
- Longitudinal propagation of the waves
- Hydrodynamic theory,
  cold collisionless plasma
- Consideration of nonresonant component



$\omega \rightarrow \omega_H$, ω_p

$\mathbf{B_w} = \mathrm{Re}[b_w \mathbf{e_+} \exp(ik_w z)]$
$\mathbf{e_+} = 2^{-1/2}(\mathbf{x_0} + i\mathbf{y_0})$

a_+ - resonant polarization
a_- - nonresonant polarization



Main equations

$$\frac{\partial \mathbf{V}_\perp}{\partial t} + \omega_H [\mathbf{V}_\perp, \mathbf{z_0}] = \frac{e}{mc}\frac{\partial \mathbf{A}_\perp}{\partial t} - \frac{e}{mc} V_\parallel [\mathbf{z_0}, \mathbf{B_w}]$$

$$\frac{\partial V_\parallel}{\partial t} = \frac{e}{m}\frac{\partial \varphi_\parallel}{\partial z} - \frac{e}{mc}[\mathbf{V}_\perp, \mathbf{B_w}]$$

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{A}_\perp = -\frac{4\pi e}{c} N_0 \mathbf{V}_\perp$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \varphi_\parallel}{\partial z}\right) + 4\pi e N_0 V_\parallel = 0$$

$$A_\pm << \frac{B_w}{k_w}$$

- Immobile ions
- Non-relativistic

$$\mathbf{A}_\perp = \mathrm{Re}\{[\mathbf{e}_+ A_+(z) + \mathbf{e}_- A_-(z)]\exp(-i\omega t)\},$$
$$\mathbf{V}_\perp = \mathrm{Re}\{[\mathbf{e}_+ V_+(z) + \mathbf{e}_- V_-(z)]\exp(-i\omega t)\},$$
$$V_\parallel = \mathrm{Re}\{V_p(z)\exp(-i\omega t)\},$$
$$\varphi_\parallel = \mathrm{Re}\{\varphi_p(z)\exp(-i\omega t)\}.$$

Equations for coupled modes

$$\left(-\frac{c^2}{\omega^2}\frac{d^2}{dz^2}-n_{0+}^2\right)A_+ = gA_-\exp(i2k_wz),$$

$$\left(-\frac{c^2}{\omega^2}\frac{d^2}{dz^2}-n_{0-}^2\right)A_- = gA_+\exp(-i2k_wz).$$

$$V_\pm = \frac{e}{mc}\left[\left(1-\frac{n_{0\pm}^2}{v}\right)A_\pm + \frac{g}{v}A_\mp\exp(\pm i2k_wz)\right],$$

$$V_p = \frac{u_w}{4(1-v)}\left[V_+\exp(-ik_wz)+V_-\exp(ik_wz)\right].$$

$A_\pm$- vector-potential

$A_\pm(z)=a_\pm(z)\exp(\pm ik_wz)$ $\Rightarrow$ ordinary differential eqs. $\Rightarrow$ $A_\pm = A_{0\pm}\exp\left(i\frac{\omega}{c}(n\pm n_w)z\right)$

$k_\pm=k\pm k_w$ $\qquad$ $n_\pm=n\pm n_w$

$$\left\{(n+n_w)^2-n_{0+}^2\right\}\left\{(n-n_w)^2-n_{0-}^2\right\}-g^2=0$$

$u=\omega_H^2/\omega^2$
$v=\omega_p^2/\omega^2$
$n_w=ck_w/\omega$

$$n_{0\pm}^2 = 1-v\frac{(1-v)\left(1\pm\sqrt{u}\right)-u_w/2}{(1-v)(1-u)-u_w},\quad g=\frac{1}{2}\frac{vu_w}{(1-v)(1-u)-u_w},\quad u_w=\frac{B_w^2}{B_0^2}$$

UIT range: $(1-v)(1-u)<u_w$

Structure of general solution

$$\mathbf{A}_\perp = \sum_{j=1}^{4} C_j \exp(-i\omega t)\left[\mathbf{e}_+ \exp(ik_{j+}z) + \mathbf{e}_- K_j \exp(ik_{j-}z)\right] =$$

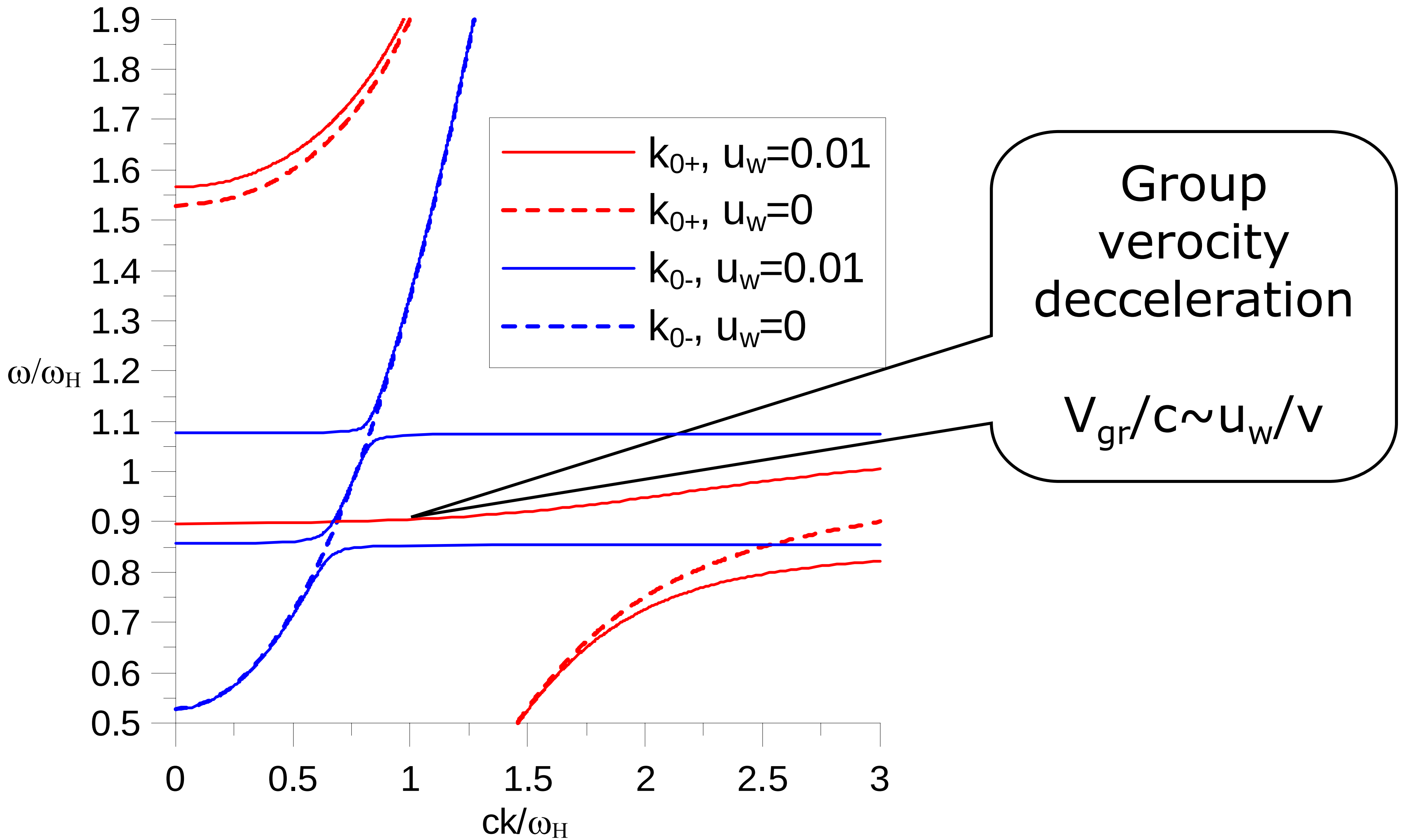
$$\sum_{j=1}^{4} C_j \exp(ik_j z - i\omega t)\left[\mathbf{e}_+ \exp(ik_w z) + \mathbf{e}_- K_j \exp(-ik_w z)\right]$$

- bicomponent normal wave

$$K_j = \frac{(n+n_w)^2 - n_{0+}^2}{g} = \frac{g}{(n-n_w)^2 - n_{0-}^2}$$

$$\frac{eE_z}{mc\omega} = -\frac{g}{\sqrt{w}} \mathrm{Re}\left[\left(a_-(1-\sqrt{u})e^{in_u z} - a_+(1+\sqrt{u})e^{-in_u z}\right)e^{-i\omega t}\right]$$

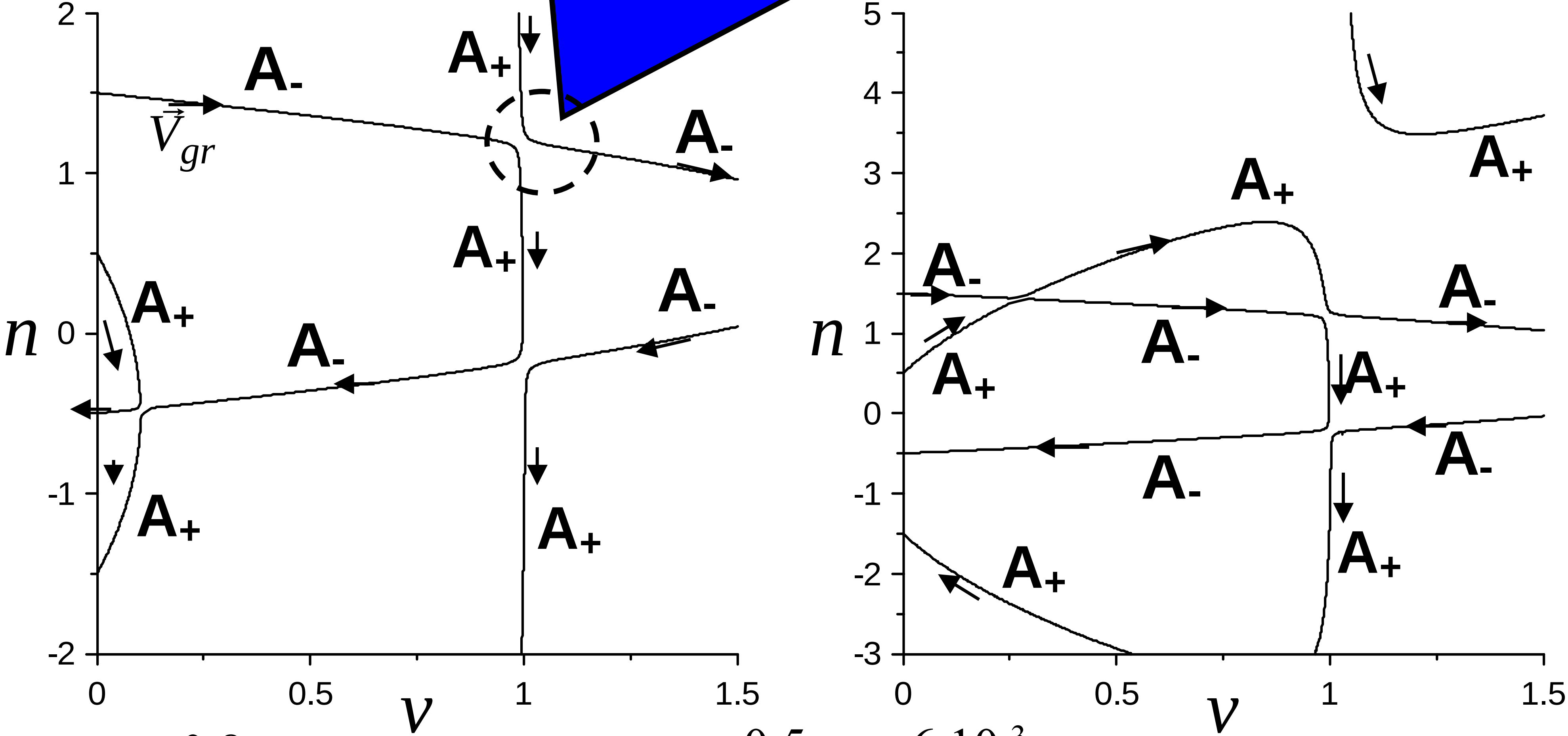
Dispersive law for $n_{0\pm}$



Mode conversion

$n_{\pm} = n \pm n_w$

$n_i \rightarrow n_{j \neq i} \Leftrightarrow n_+ \rightarrow n_- + 2n_w \Leftrightarrow n_{o+} \rightarrow n_{o-} + 2n_w$



u=0.8

$n_w = 0.5$, $u_w = 6 \cdot 10^{-3}$

u=1.2

Conversion efficiency

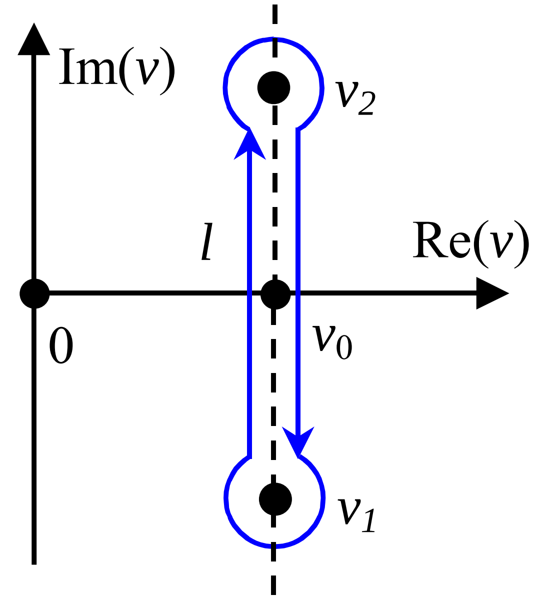
T~exp(–δ)

$$\delta = \left| \frac{\omega}{c(dv/dz)} \oint_l \frac{n_1 - n_2}{2} dv \right|$$

$$\left\{(n+n_w)^2 - n_{0+}^2\right\}\left\{(n-n_w)^2 - n_{0-}^2\right\} - g^2 = 0$$

approximation by 2 hyperbolas

$$A + B(v - v_0) + C(n - n_0)^2 + D(n - n_0)(v - v_0) + E(v - v_0)^2 = 0$$



$$\delta \approx \frac{2\pi\omega}{c\left|dv/dz\right|} \frac{1+\sqrt{u}}{8(N_0 - n_w)} \frac{u_w^2}{(1-v_0)(1-u) - u_w} \propto \frac{u_w^2}{(1-v_0)(1-u) - u_w} L$$

For inhomogeneity scale L > 5(c/ω) : T < 50%

(n_w=1.6, u_w=0.04, u=1)

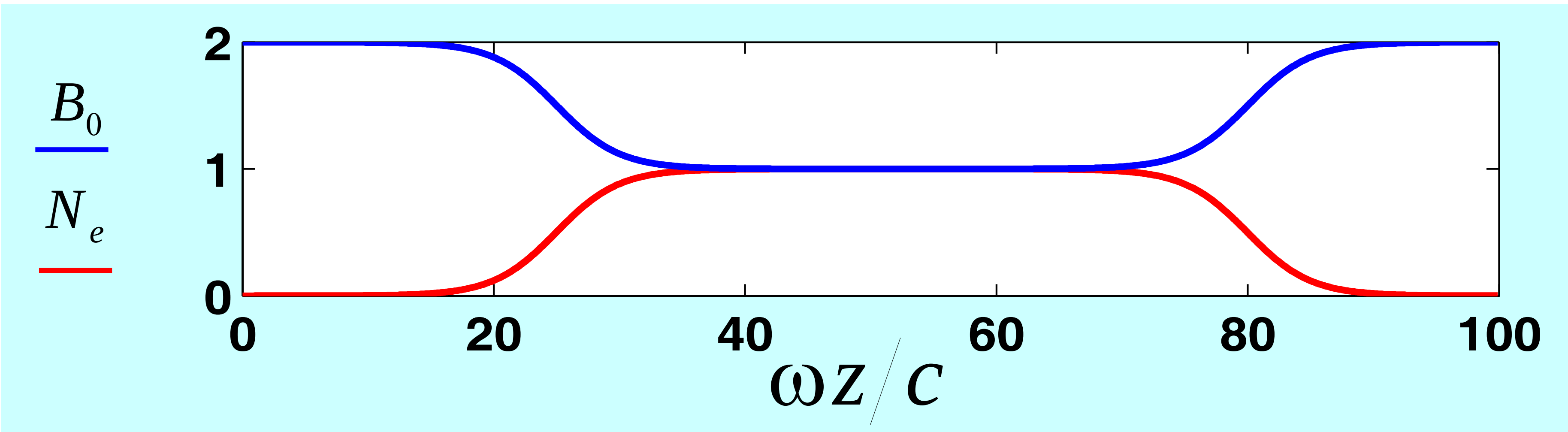
Transformation for $\lambda_w >> c/\omega$:

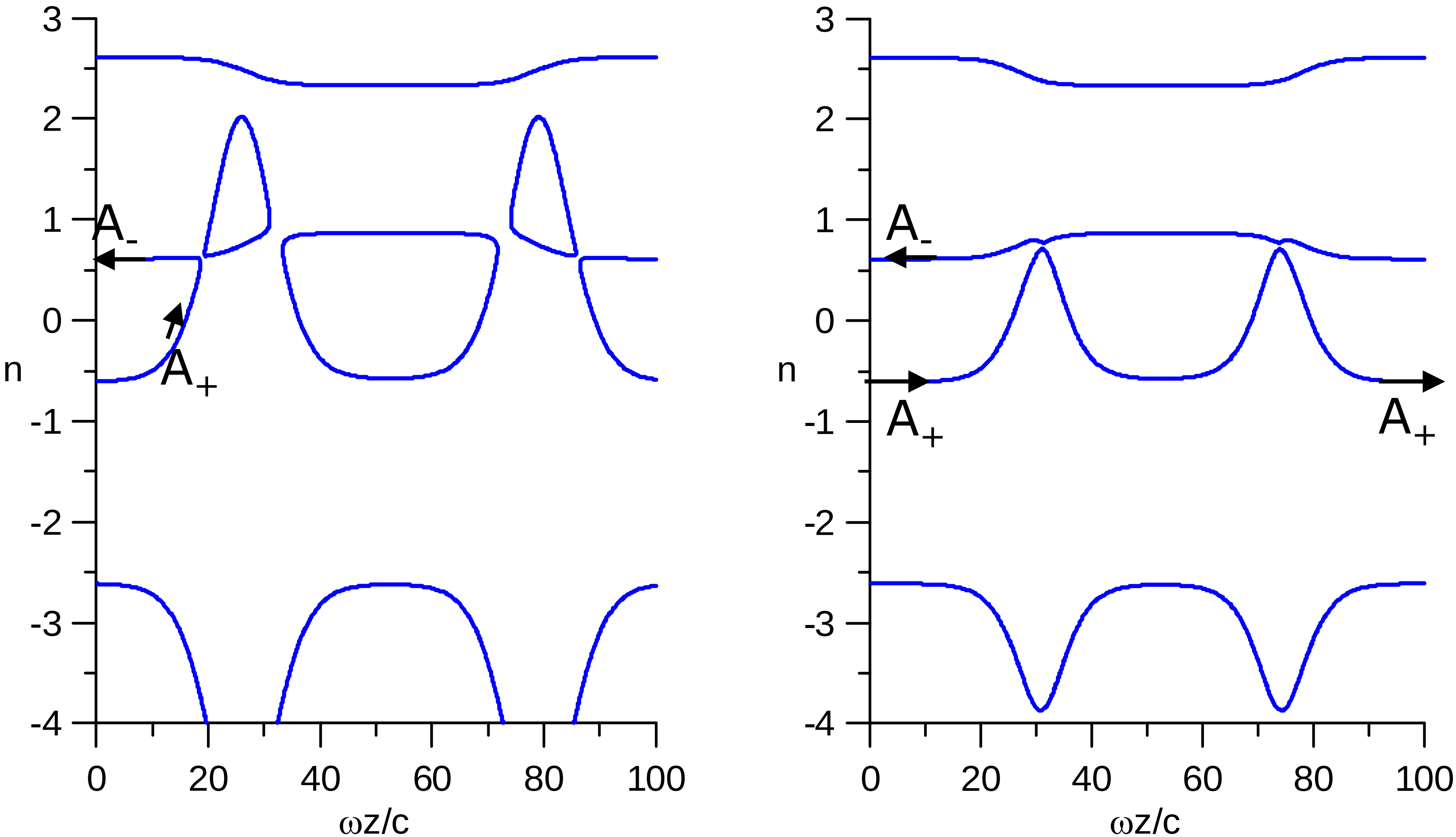
V.V.Kocharovsky, Vl.V.Kocharovsky // Plas.Phys.Reports, 1980

Density and external magnetic field profiles

$$n_{0+}^2 = 1 - v\frac{(1-v)(1+\sqrt{u}) - u_w/2}{(1-v)(1-u) - u_w} \qquad v \le 1$$

$$n_{0+}^2 > 0 \Leftrightarrow (1-v)(1-u) - u_w < 0 \quad \Rightarrow \quad u \ge 1$$





u=const=1

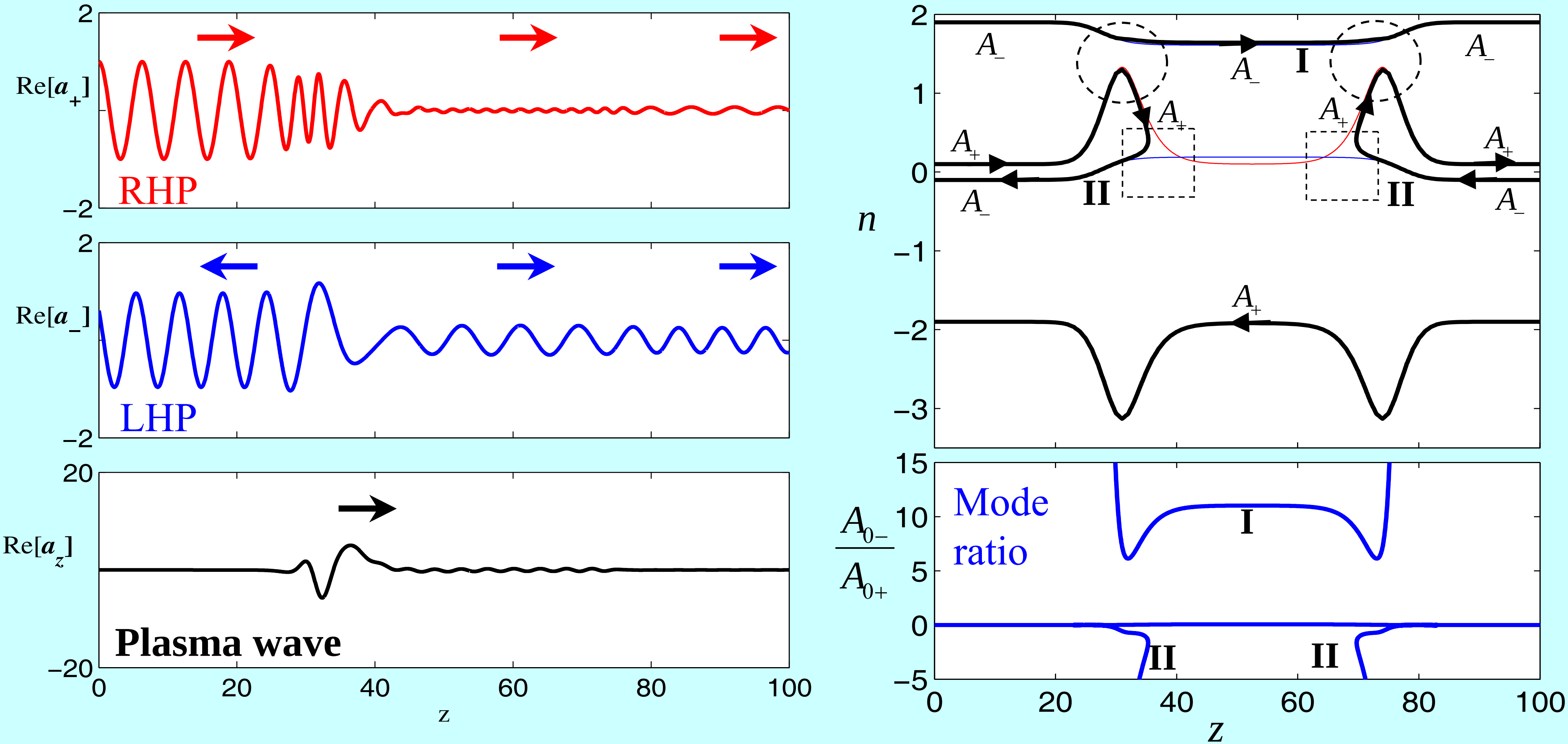
u=u(z)

n_w=1.6, u_w=0.04, v=0.99

Opaque regime

n_u=0.9

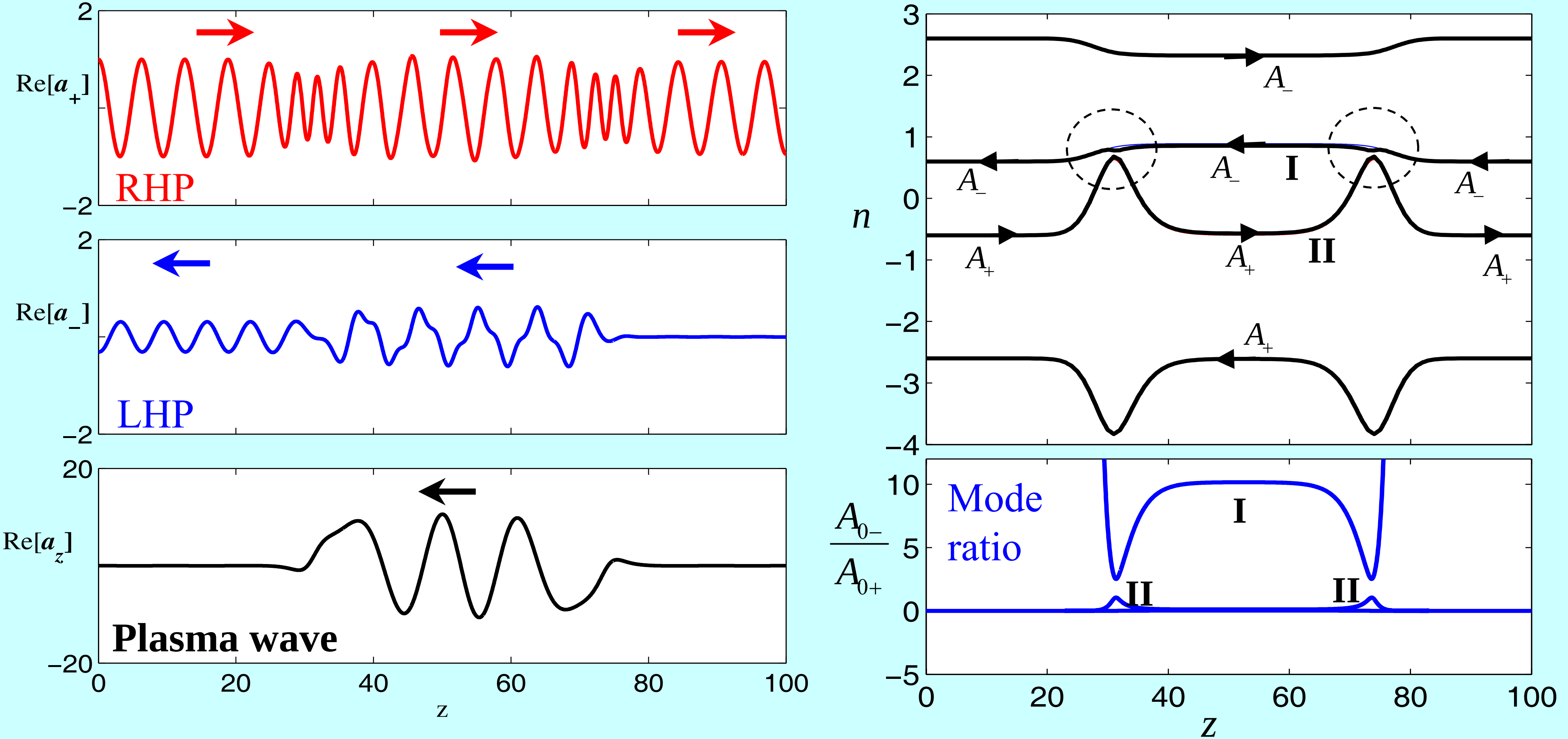
No propagation for RHP mode



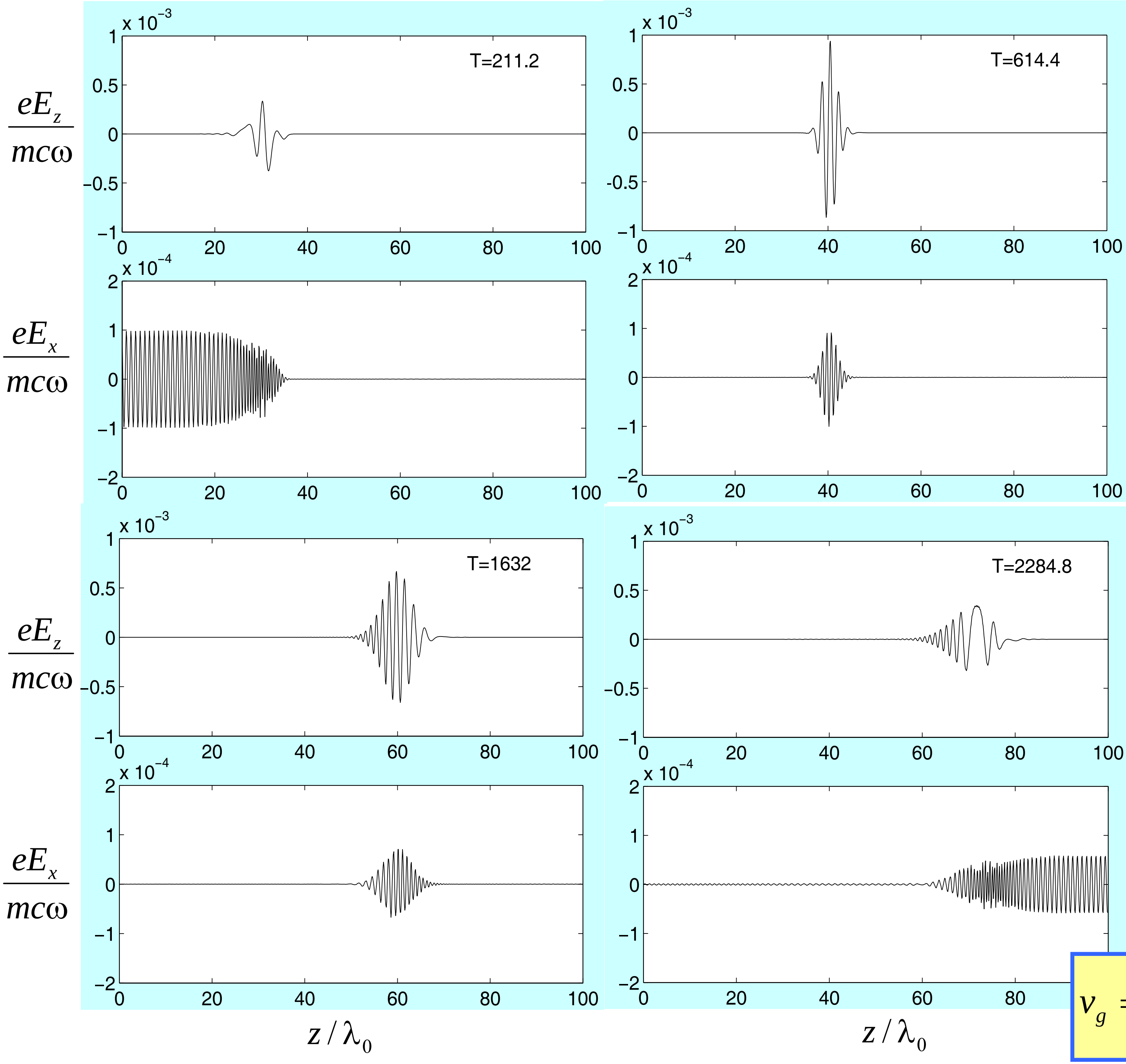
u_w=0.04, v_0=0.99, u_0=1

UIT regime

n_w=1.6



u_w=0.04, v_0=0.99, u_0=1



$n_w = 1.6$

$u_w = 0.04$

$u_0 = 1$

$v_0 = 0.99$

$$f(t) = a_0 \exp\left[-\left(\frac{t - t_0}{\tau}\right)^2\right]$$

$a_0 = 0.0001$

$\tau = 100T_0$

$t_0 = 200T_0$

(RHCP)

Group velocity estimation :

$$v_g = \frac{2w_0}{v_0}c \approx 0.02c$$

$$v_g = \frac{60\lambda_0 - 40\lambda_0}{1632T_0 - 614.4T_0} \approx 0.02c$$

Main results

- **Effect of UIT is investigated with self-consistent consideration of polarization of electromagnetic waves**
  - The structure of bicomponent normal waves is investigated theoretically (the dispersive equation is obtained; the mode conversion effect is considered for inhomogeneous plasma)
  - We propose the profiles of density and external magnetic field, most suitable for the experimental implementation of the UIT effect.
  - Numerical calculations proved, that the proper selection of undulator wavelength combined with judicious choice of axial magnetic field profile enables microwaves to penetrate plasma with realistic (smooth) density profiles at moderate level of the undulator field.