

For the transmission of high power millimeter-waves, the mode content (or transverse wave number spectrum) of paraxial wave beams must often be altered in order to match them to waveguides or quasi-optical transmission lines. This is commonly achieved by means of Yeti-footprint mirrors (YFMs, a.k.a. non-quadratic mirrors), which are synthesized with the Katzenelenbaum-Semenov-Algorithm (KS-Algorithm) [1]. This algorithm yields an ideal phase corrector which must be transformed into a smooth machineable surface in order to form a mirror. The poster describes a method to produce an easily machineable mirror surface which produces little stray radiation and has low arcing risk.

1 Introduction

Due to the cyclic nature of the phase, the gradient of an ideal phase corrector synthesized by the KS-algorithm has a curl which produces unresolvable 2π -jumps on the mirror surface. They occur at residual points which are similar to screw dislocations in crystalline materials. When these 2π -jumps are machined with a cutter head of finite radius, the quality of the phase corrector deteriorates and stray radiation will occur, which is not desirable especially inside gyrotrons. If those 2π -jumps are machined with a high accuracy, however, arcing or hot spots may occur as there will be sharp edges on the mirror surface. In addition, the mirror becomes more expensive. Therefore a method will be described in the following which diminishes or avoids these 2π -jumps at the cost of conversion efficiency, while still suppressing non-paraxial components of the mode spectrum (i.e. stray radiation).

2 Synthesis of YFMs

The synthesis of YFMs is a well established technique in the transmission of high power microwaves. It was first described for general phase correctors in [1]. The application to microwave mirrors was first demonstrated in [2]. The algorithm is shown in Figure 1.

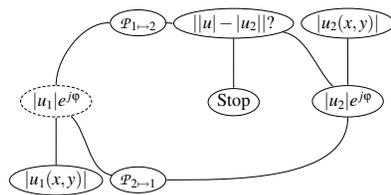


Figure 1: The Katzenelenbaum-Semenov-Algorithm.

The field is propagated back and forth between two apertures while imposing the given amplitude in the input aperture, and the desired amplitude distribution in the target aperture. After some iterations we have an input and target distribution with an amplitude distribution that is close to the given input and desired target distribution respectively. The difference in the phase distributions is compensated by two phase correctors, which are

mirrors in our case.

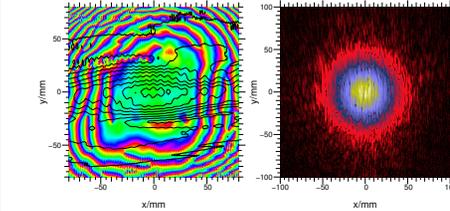


Figure 2: Input distribution with corresponding phase corrector (left) and target distribution (right).

As an example, Figure 2 shows such a phase corrector (color) together with the incident field (isolines) on the left side, and the achieved target amplitude distribution (the goal was Gaussian distribution) with ideal phase correctors. "Ideal" means that the height of the mirror surface corresponds exactly to the calculated phase corrector. However such a surface can not be manufactured because there are points on the mirror surface where a closed integral around them yields a phase difference of 2π . That means, somewhere on this path there must be a 2π -jump (which is no phase jump).

The following analogy helps to understand this problem: Imagine a few infinitely thin parallel wires which carry different DC currents. In a plane perpendicular to these wires we see a 2D magnetic field. If we choose a region of this plane in which no wire is located, the magnetic field can be represented as the gradient of a scalar potential. The integration of this gradient yields a smooth surface (i.e. the scalar potential). If there is one or more wires in the selected region, the field has an additional curl. The wires represent singular points. If we integrate the magnetic field around those points, we obtain the current in the wire. If we would switch off the wires inside our region, the curl would vanish and the field would be integrable again. The scalar potential that we find after the integration of the gradient corresponds to our smooth mirror surface. Another analogy are screw dislocations in crystalline materials.

The angular direction of such a 2π -jump can be disregarded for now, we can just say that it originates in such a residual point. Hence, the mirror will be something like a Fresnel lens which is not desirable for the above reasons. Figure 3 shows those residual points and phase jumps on a real mirror surface.



Figure 3: Residual points with phase jumps on a mirror surface.

3 Modification of the KS-Algorithm

The mentioned curl in the gradient of the phase corrector is inconsistent with the requirement of a smooth mirror surface. Therefore sophisticated algorithms or simply experience and craftsmanship are required to form a mirror surface out of a phase

corrector, i.e. adding $n \cdot 2\pi$ to each point of the phase corrector to make the mirror surface as smooth as possible. Simply removing the curl (see [3]) often degrades the result too much, keeping it results in 2π -jumps. In most cases the 2π -jumps are shifted to locations with a low field amplitude (if possible) and smoothed to fit the radius of the cutter head.

The KS-algorithm described so far has the two amplitude distributions as the only constraints. The idea is now to introduce the additional requirement of a curl-free phase corrector in conjunction with the input phase distribution. This is achieved by altering the dashed box in Figure 1. It can actually be written as $u_1 e^{j\phi} \cdot e^{j(\phi - \phi)}$ where ϕ_i is the impinging phase distribution and the third factor is the phase corrector. For the classical KS-Algorithm ϕ_i vanishes. Now we replace the phase corrector by $e^{j\mathcal{R}(\phi - \phi)}$ where \mathcal{R} is the operator for the removal of the curl [3]. Now the impinging phase distribution is taken into account and a curl-free phase corrector is imposed.

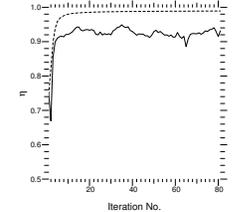


Figure 4: Convergence of the classical (dashed) and modified (solid) KS-Algorithm.

Figure 4 shows the convergence of the classical and modified KS-algorithm for the problem of Figure 2. Here, η is the normalized scalar product of the desired and the achieved target amplitude distribution (1 for perfect agreement, 0 for orthogonality). One can clearly see that the modified version does not converge as well and to lower conversion efficiencies when compared to the classical version. The result looks accordingly, see Figure 5.

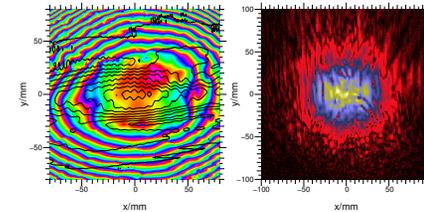


Figure 5: Input distribution with corresponding phase corrector (left) and target distribution (right) for the modified KS-Algorithm.

What is more interesting in Figure 5 is the fact that there are little or no residual points in the phase corrector. That means we can produce a mirror surface with no 2π -jumps. The achieved field distributions in Figures 2 and 5 are obtained by the ideal phase correctors resulting from the classical and modified KS-algorithm respectively. Obviously, the phase correctors must be "unrolled" and smoothed, i.e. one must add $n \cdot 2\pi$ to each point in order to obtain a flat mirror which can be machined by a cutter head with a predefined radius. The result after this procedure

with a cutter head radius of 5 mm can be seen in Figure 6.

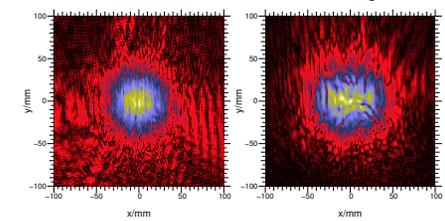


Figure 6: Achieved distribution with the classical (left) and the modified (right) KS-Algorithm.

We see that a lot of stray radiation is produced in the result of the classical KS-algorithm due to the smoothing process, while the result for the modified version is practically unchanged since there are no residual points in it. The η for the classical KS-algorithm in Figure 4 degrades to about the same value as for the modified version (in other examples it might even get lower) while it is practically unchanged in the modified case.

But the mere η is not the only measure of quality. It is often more important that the full beam power hits the next mirror or window. So in the final assessment of our example, the modified version wins in spite of the initially lower η because the beam remains paraxially.

4 Summary

It has been shown that the removal of phase jumps in the synthesis of Yeti footprint mirrors strongly reduces the risk of arcing and the amount of the generated stray radiation. This is in most cases achieved at the cost of conversion efficiency (not in all cases since the final η counts). In order to nevertheless obtain a high conversion efficiency, the number of mirrors could be increased (mirror triplets or quadruplets instead of pairs).

For the conversion of arbitrary mode spectra, at least mirror pairs are needed. In this case, the removal of the curl in the target plane(s) would be an option and will be subject to further investigation.

References

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- [3] Michel, G., Thumm, M., *Spectral Domain Techniques for Field Pattern Analysis and Synthesis*, Surveys on Mathematics for Industry (1999) **8**: 259–270