Ohmic Losses in Coaxial Gyrotron Cavities with Corrugated Insert

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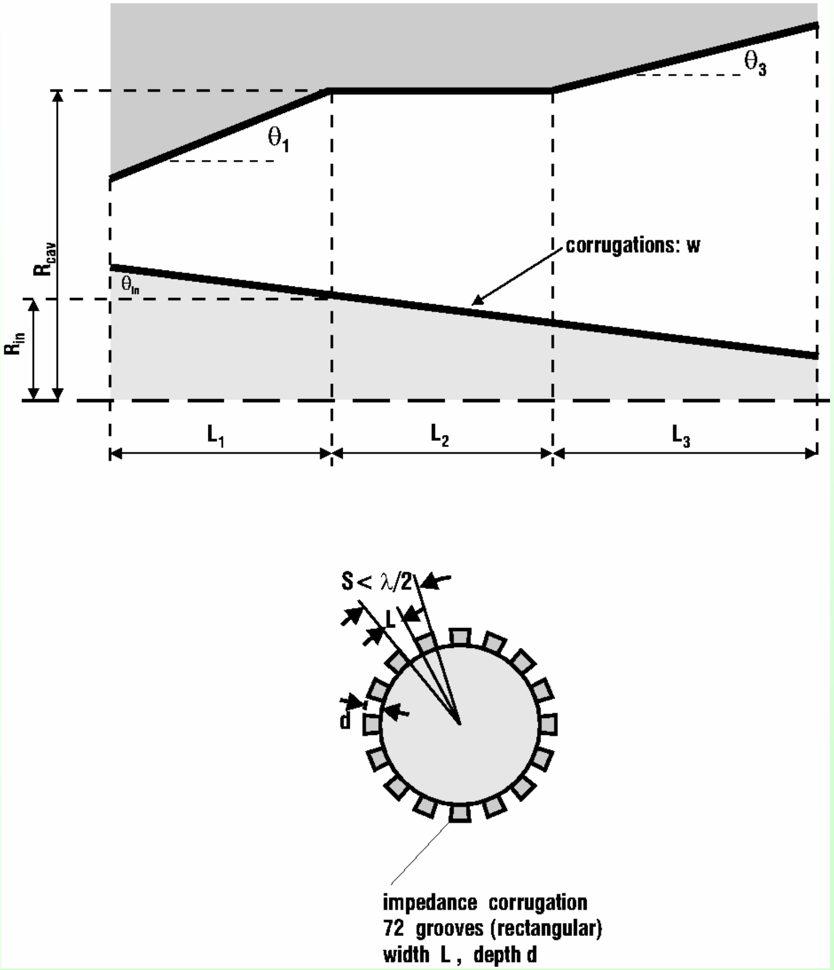
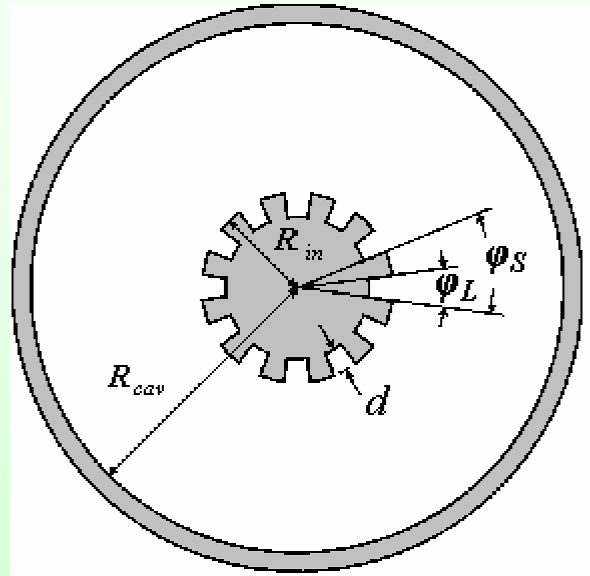
Introduction





The transverse cross-section of the coaxial gyrotron cavity.

General expressions for losses in gyrotron resonators

Local density of ohmic losses:

$$\rho = \frac{c}{16\pi}(\delta k)|H_\tau|^2 \qquad (1)$$

Here δ is the skin depth, k is the free-space wave number, c is the velocity of light in vacuum, H_τ is the tangential component of the magnetic field at the local point on the surface.

The main contribution to H_τ comes from the longitudinal magnetic field:

$$H_z = Af(z)u(r_\perp)\exp(im\varphi) \qquad (2)$$

$u(r_\perp)$ is the so-called membrane function. Using the energy balance equation

$$\frac{\omega}{16\pi}\int_V \left\{\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right\} dV = P_{out} Q_{dif} \qquad (3)$$

General expressions for losses in gyrotron resonators

and the normalization of the membrane function $$\int_{S_\perp} |u(r_\perp)|^2 ds_\perp = \frac{1}{k^2} \qquad \textbf{(4)}$$

one obtains

$$|A|^2 = \frac{8\pi}{c} \frac{k Q_{dif} P_{out}}{\int_0^{z_{out}} |f(z)|^2 dz} \qquad \textbf{(5)}$$

$$\rho(z) = \frac{1}{2} \delta k^2 \frac{|f(z)|^2}{\int_0^{z_{out}} |f(z)|^2 dz} |u(r_\perp)|^2 Q_{dif} P_{out} \qquad \textbf{(6)}$$

General expressions for losses in gyrotron resonators

$$\rho_{in}^{top}(z) = \rho_0(z)\left\langle |u(r_\perp)|^2 \right\rangle_{top} \quad \textbf{(7)}$$

$$\rho_{in}^{bot}(z) = \rho_0(z)\left\langle |u(r_\perp)|^2 \right\rangle_{bot} \quad \textbf{(8)}$$

$$\rho_{in}^{side}(z) = \rho_0(z)\left\langle |u(r_\perp)|^2 \right\rangle_{side} \quad \textbf{(9)}$$

$$\overline{\rho}_{in}(z) = \rho_0(z)\left\langle |u(r_\perp)|^2 \right\rangle_{per} \quad \textbf{(10)}$$

$$\rho_0(z) = \rho(z)/|u(r_\perp)|^2$$

$$\left\langle |u(r_\perp)|^2 \right\rangle_{top} = \frac{2}{\varphi_S - \varphi_L}\int_{\varphi_L/2}^{\varphi_S/2} |u(r_\perp)|^2\Big|_{r=R_{in}} d\varphi$$

$$\left\langle |u(r_\perp)|^2 \right\rangle_{side} = \frac{1}{d}\int_{R_{in}-d}^{R_{in}} |u(r_\perp)|^2\Big|_{\varphi=\varphi_L/2} dr$$

$$\left\langle |u(r_\perp)|^2 \right\rangle_{bot} = \frac{1}{\varphi_L}\int_{-\varphi_L/2}^{\varphi_L/2} |u(r_\perp)|^2\Big|_{r=R_{in}-d} d\varphi$$

$$\left\langle |u(r_\perp)|^2 \right\rangle_{per} = \frac{1}{\varphi_S}\times\left((\varphi_S-\varphi_L)\left\langle |u(r_\perp)|^2 \right\rangle_{top} + \varphi_L\left\langle |u(r_\perp)|^2 \right\rangle_{bot} + \frac{2d}{R_{in}}\left\langle |u(r_\perp)|^2 \right\rangle_{side} \right),$$

$$\varphi_S = 2\pi/N \qquad \varphi_L = (L/S)\varphi_S$$

$u(r_\perp)$???

Surface impedance model (SIM)

SIM is based on the assumption that the width of the corrugations is smaller than half of the wavelength of oscillations ($L < \lambda/2$). In this approximation the fields can be assumed homogeneous inside the grooves, although they can vary from one groove to the next groove according to the azimuthal wave number. Here the groove is considered as a part of rectangular waveguide, and the fields can be approximated by a part of a rectangular $TE_{1,0}$ mode. In SIM the eigenvalue χ_{mp} of the TE_{mp} mode can be determined by means of the following transcendental equation:

$$J'_m(\chi_{mp})\left[N'_m\left(\frac{\chi_{mp}}{C}\right)+WN_m\left(\frac{\chi_{mp}}{C}\right)\right]-N'_m(\chi_{mp})\left[J'_m\left(\frac{\chi_{mp}}{C}\right)+WJ_m\left(\frac{\chi_{mp}}{C}\right)\right]=0 \quad \textbf{(11)}$$

J_m and N_m are the Bessel and Neumann functions respectively, $C = R_{cav}/R_{in}$, R_{cav}, is the outer radius of the cavity, R_{in} is the insert radius, and W is the corrugation parameter defined as:

$$W = \frac{L}{S}\tan\left(\frac{2\pi}{\lambda}d\right) \quad \textbf{(12)}$$

Surface impedance model (SIM)

$$\rho_{in}^{top}(z)=\frac{2\pi\delta}{\lambda^2}\cdot\frac{|f(z)|^2}{\int_0^{z_{out}}|f(z)|^2 dz}\cdot\frac{Q_{dif}P_{out}}{(\chi_{mp}^2-m^2)R_Z-\frac{\chi_{mp}^2}{C^2}(1+W^2)+\frac{\chi_{mp}}{C}\left[W+\frac{\chi_{mp}d}{R_{cav}}\left(\frac{L}{S}+\frac{S}{L}W^2\right)\right]+m^2} \quad \textbf{(13)}$$

$$\rho_{in}^{bot}(z)=\rho_{in}^{top}(z)\left(1+\frac{S^2}{L^2}W^2\right) \quad \textbf{(14)}$$

$$\rho_{in}^{side}(x,z)=\rho_{in}^{bot}(z)\cos^2\left(\frac{\chi_{mp}}{R_{cav}}x\right) \quad \textbf{(15)}$$

$$\bar{\rho}_{in}(z)=\rho_{in}^{top}(z)\left[1+\frac{S}{L}W^2+\frac{R_{cav}W}{\chi_{mp}L}+\frac{d}{S}\left(1+\frac{S^2}{L^2}W^2\right)\right] \quad \textbf{(16)}$$

$$R_Z=\left[N_m^{'}(\chi_{mp}/C)+WN_m(\chi_{mp}/C)\right]^2/\left(C^2\left[N_m^{'}(\chi_{mp})\right]^2\right)$$

Singular integral equation (SIE)

In the region ($R_{in} < r < R_{cav}$) the membrane function can be expressed in terms of superposition of spatial harmonics:

$$u \equiv u^{+} = \sum_{n=-\infty}^{\infty} A_n f_n(r) \exp(ik_n\varphi) \qquad \textbf{(17)}$$

where, $f_n(r) = G_{k_n}(\chi, \chi/C, \chi r/R_{cav})$ $\qquad G_\nu(a,b,c) = \left(J'_\nu(a)N_\nu(c) - N'_\nu(a)J_\nu(c)\right)/\left(J'_\nu(a)N'_\nu(b) - N'_\nu(a)J'_\nu(b)\right)$

$$k_n = m + nk_S \qquad k_S = 2\pi/\varphi_S = N$$

Due to quasi-periodicity of the membrane function with respect to φ one can consider only the interval ($-\varphi_S/2, \varphi_S/2$). In grooves the membrane function can be expressed in terms of the Fourier series:

$$u \equiv u^{-} = \sum_{n=0}^{\infty} X_n g_n(r) \cos\left(\xi_n(\varphi + \varphi_L/2)\right) \qquad \textbf{(18)}$$

where $g_n(r) = G_{\xi_n}(\chi/C', \chi/C, \chi r/R_{cav})$ **Introducing the function,** $F(\varphi) = (R_{cav}/\chi)\partial u^{+}/\partial r\big|_{r=R_{in}}$ **one obtains from (17) the expression**

$$F(\varphi) = \sum_{n=-\infty}^{\infty} A_n \exp(ik_n\varphi) \qquad \textbf{(19)}$$

Singular integral equation (SIE)

Matching u^+ and u^- and their r-derivatives at the interface between the waveguide and the groove $\varphi \in (-\varphi_L/2, \varphi_L/2), r = R_{in}$

one obtains the relations

$$\sum_{n=-\infty}^{\infty} A_n f_n(R_{in}) \exp(ik_n\varphi) = \sum_{n=0}^{\infty} X_n g_n(R_{in}) \cos(\xi_n(\varphi + \varphi_L/2)) \quad (20)$$

$$F(\varphi) = \sum_{n=0}^{\infty} X_n \cos(\xi_n(\varphi + \varphi_L/2)) \quad (21)$$

Making inverse Fourier transforms of (19) and (21), one can express unknown amplitudes of spatial and Fourier harmonics in terms of integrals of $F(\varphi)$:

$$A_n = \frac{1}{\varphi_S} \int_{-\varphi_L/2}^{\varphi_L/2} F(\theta) \exp(-ik_n\theta) d\theta \quad (22)$$

$$X_n = \frac{2\varepsilon_n}{\varphi_L} \int_{-\varphi_L/2}^{\varphi_L/2} F(\theta) \cos(\xi_n(\theta + \varphi_L/2)) d\theta \quad (23)$$

where $\varepsilon_n = \begin{cases} 1/2, & n = 0, \\ 1, & n \neq 0. \end{cases}$

Singular integral equation (SIE)

Substituting (22) and (23) into (20), one obtains the integral equation

$$\int_{-\varphi_L/2}^{\varphi_L/2} H(\varphi,\theta)F(\theta)d\theta = 0 \qquad -\varphi_L/2 < \varphi < \varphi_L/2 \qquad (24)$$

where $H(\varphi,\theta) = G_1(\varphi-\theta) + G_2(\varphi-\theta) + G_2(\varphi+\theta+\varphi_L)$, $G_1(x) = (1/\varphi_S)\sum_{n=-\infty}^{\infty} W_{k_n}(\chi,\chi/C)\exp(ik_n x)$ $G_2(x) = -(1/\varphi_L)\sum_{n=0}^{\infty} \varepsilon_n W_{\xi_n}(\chi/C',\chi/C)\cos(\xi_n x)$

$W_\nu(a,b) = G_\nu(a,b,b)$

The kernel of the integral equation (24) $H(\varphi,\theta)$ has a logarithmic singularity at $\varphi = \theta$. The first kind integral equation with a logarithmic singularity is an ill posed problem. Therefore its direct numerical analysis is difficult. However, the integral equation (24) can be easily reduced to the singular integral equation (the integral equation with the Cauchy-type singularity) with an additional condition, for which effective direct numerical methods of solution have been elaborated. Differentiating (24), one obtains the desired singular integral equation

$$\int_{-\varphi_L/2}^{\varphi_L/2} H'(\varphi,\theta)F(\theta)d\theta = 0 \qquad -\varphi_L/2 < \varphi < \varphi_L/2 \qquad (25)$$

where the prime means differentiation with respect to φ. Integrating one finds the desired additional condition

$$\int_{-\varphi_L/2}^{\varphi_L/2} M(\theta)F(\theta)d\theta = 0 \qquad (26)$$

where $M(\theta) = \int_{-\varphi_L/2}^{\varphi_L/2} H(\varphi,\theta)d\varphi$

Numerical example: 2 MW, CW, 170 GHz coaxial cavity gyrotron for ITER

This coaxial cavity gyrotron operates in the $TE_{34,19}$ mode. The quality factor of its resonator is $Q_{dif} = 1642$. The skin depth is given by the expression $\delta = \sqrt{\lambda / \pi Z_0 \sigma}$ where $Z_0 = 377\Omega$ is the free space wave impedance and σ is the electrical conductivity. It is assumed that $\sigma = 5.7 \cdot 10^4 (\Omega \cdot mm)^{-1}$ which is the heat conductivity of ideal copper at room temperature and that the generated RF power in the cavity is 2.2 MW.

Table I. Ohmic losses in the insert in the middle of the cavity, where the absolute value of the normalized RF field is unity.

density (kW/cm^2)	SIM	SIE
ρ_{in}^{top}	~0	0.009
ρ_{in}^{side}	0.024	0.010
ρ_{in}^{bot}	0.048	0.019
$\overline{\rho_{in}}$	0.057	0.027

Numerical example: 2 MW, CW, 170 GHz coaxial cavity gyrotron for ITER

For the $TE_{34,19}$ coaxial gyrotron cavity ($d \sim \lambda / 4$), calculations based on SIE predict significantly lower losses in the corrugated surface in comparison with SIM. As an important practical consequence of this result, it follows that R_{in} can be increased from its present value 8.00 mm to 8.20 mm without overheating the insert. Indeed, within the SIM formalism such an increase would lead to

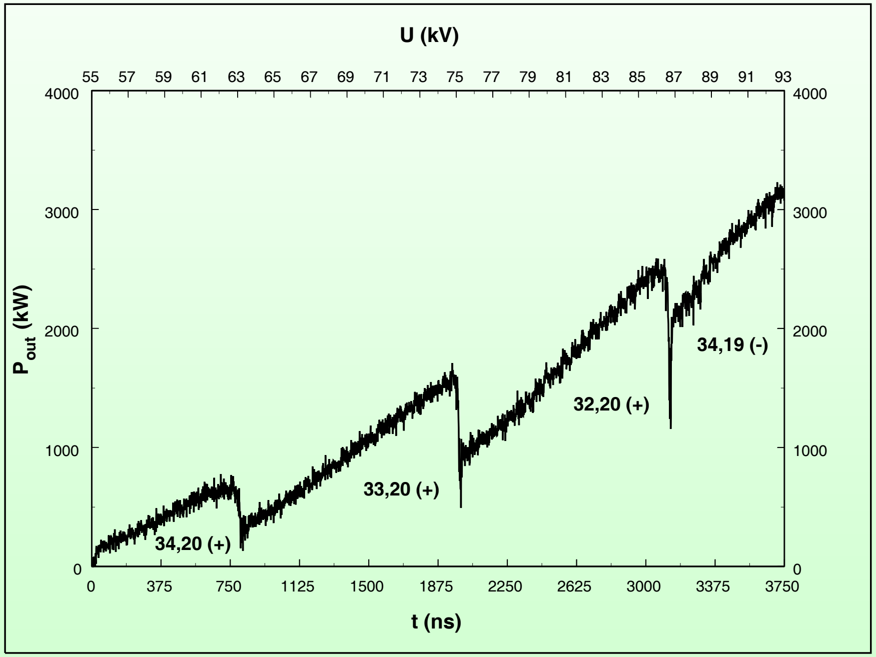
$\overline{\rho_{in}} = 0.06\,kW / cm^2 \rightarrow 0.12\,kW / cm^2$ **which can not be tolerated, while with SIE formalism we obtain**

$\overline{\rho_{in}} = 0.03\,kW / cm^2 \rightarrow 0.06\,kW / cm^2$ **which is well below the tolerable limit of $\overline{\rho_{in}} = 0.1\,kW / cm^2$. On the other hand, it could be expected that the thicker insert (by 0.2 mm) would significantly contribute to reduction of mode competition, because the quality factor of the operating mode would decrease only slightly, while the quality factors of parasitic modes would become much smaller.**

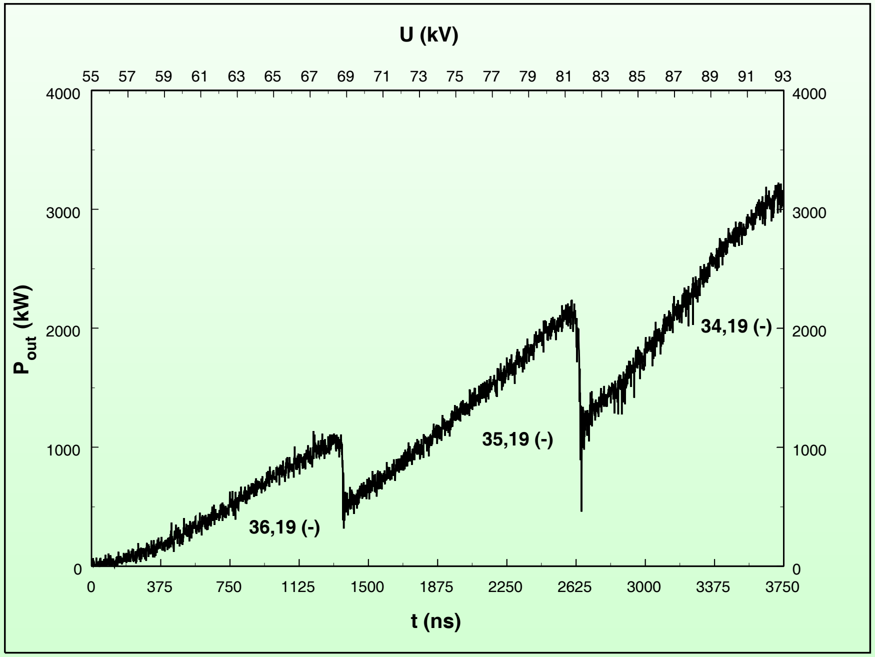
Numerical example: the 2 MW, CW, 170 GHz coaxial cavity gyrotron for ITER

mode	Q (R_{in}=8.0 mm)	Q (R_{in}=8.2 mm)	reduction (%)
34,19	1642	1606	2.2
35,20	1753	1662	5.2
37,19	1767	1762	0.3
34,20	1679	1602	4.6
36,19	1728	1718	0.6
33,20	1582	1467	7.3
35,19	1688	1668	1.2
32,20	1455	1304	10.4
31,20	1297	1123	13.4
33,19	1584	1526	3.7
35,18	1621	1614	0.4
60,42	83999	69830	16.9
57,41	32317	9388	71.0

Numerical example: 2 MW, CW, 170 GHz coaxial cavity gyrotron for ITER



R_{in}=8.0 mm



R_{in}=8.2 mm

Conclusions

1) The importance of inclusion of higher spatial harmonics in evaluating ohmic losses in a corrugated insert of a coaxial gyrotron cavity has been demonstrated.

2) This can be achieved by means of the method of the singular integral equation.

3) For the coaxial cavity gyrotron for ITER ohmic losses predicted by SIE are twice as small as predicted by SIM.

4) This allows one to increase the radius of the insert by 0.2 mm and to improve the mode competition scenario.