EFFECT OF ELECTROMAGNETICALLY INDUCED TRANSPARENCY FOR THE PROBE WAVE AT UPPER-HYBRID RESONANCE

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The parametric effect of electromagnetically induced transparency (EIT) is considered for the quasi-transversal propagation of extraordinary electromagnetic wave at upper-hybrid resonance. We investigated the possibility of wave propagation through the region of opaque plasma in EIT regime in smoothly inhomogeneous plasma (from the cut-off region near the upper-hybrid resonance to the vacuum and vice versa).

1. Introduction

Recently the significant interest is shown in the effect of electromagnetically induced transparency (EIT) in plasma [1-5]. This effect is the classical analogue of EIT in quantum systems [6] and manifests itself as formation of a "transparency window" within the zone of resonance absorption in the presence of high-power pumping wave, which is accompanied, at the same time, by extremely small group velocity of the probe wave. The typical feature of EIT in plasma is the excitation of electrostatic oscillations by the beating between pump and probe waves. The classical analogues of this effect were investigated in magnetoactive plasma in [1,2,4,5] (cold plasma) and in [3] (high-temperature plasma) for the longitudinal propagation of the waves relatively to the external magnetic field.

One of the possible applications of this effect in plasma can be diagnostics in the controlled nuclear fusion, when the probe wave (e.g. spontaneous plasma emission) is transferred from the center of tokamak to the vacuum through the cutoff region [1,3]. But in toroidal systems it is difficult to provide the longitudinal propagation of the waves. Therefore, to investigate the possible applications of this effect in the physics of tokamak plasma we considered the quasi-transversal propagation of the probe wave in cold magnetoactive plasma.

2. Basic equations

Let us consider two electromagnetic extraordinary waves (probe and pump ones) propagating in cold collisional magnetized plasma. For simplicity we will assume, that their wavevectors and the external magnetic field $\mathbf{H}=H\mathbf{z_0}$ lie in the same plane x0z (Cartesian axes are used), as it is shown in Fig. 1:

$$\mathbf{E}(x,z,t)=\sum_{j=1}^{2}\mathrm{Re}\left\{\mathbf{E_j}\exp\left(-i\omega_j t+ik_{jx}x+ik_{jz}x\right)\right\}. \qquad (1)$$

Here, $\mathbf{E}(x,z,t)$ is total electric field of propagating bichromatic wave, indexes "1" and "2" denote the probe and pump wave respectively. As it was shown in [1,3], for observation of the EIT it is essential to provide the effective excitation of plasma electrostatic mode by the beating between the probe and pump waves. In this work we will also require the fulfillment of this condition:

$$\left|\omega_L-\omega_p\right| << \omega_L,\omega_p\,, \qquad (2a)$$

$$\left|k_{Lx}\right|=\left|k_1\sin\Theta_1-k_2\sin\Theta_2\right| << \left|k_{Lz}\right|=\left|k_1\cos\Theta_1-k_2\cos\Theta_2\right|. \qquad (2b)$$

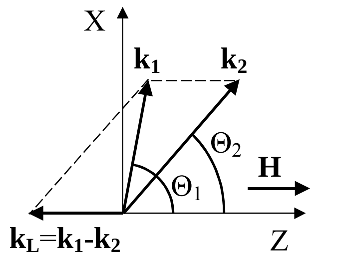

Fig. 1. Orientation of wavevectors and external magnetic field.

Here, $\Theta_{1,2}$ is the angle of wave propagation relatively to the external magnetic field, $\omega_L=\omega_1-\omega_2$ and $\mathbf{k_L}=\mathbf{k_1}-\mathbf{k_2}$ are the beatwave frequency and wavevector, $\omega_p=(4\pi e^2N_e/m)^{1/2}$ is the electron plasma frequency, N_e is the electron density, e and m are the electron charge and mass ($e>0$). In the most interesting case of quasi-transversal propagation of probe wave the angle Θ_2 should not be equal to 0 and $\pi/2$, as it follows from Eg. (2b).

We will use the hydrodynamic theory. The oscillations of the electron velocity $\mathbf{V}$ are described by the Euler equations including the Lorentz force from the wave fields:

$$\frac{\partial\mathbf{V}}{\partial t}+\omega_H\left[\mathbf{V},\mathbf{z_0}\right]+\gamma\mathbf{V}+V_x\frac{\partial\mathbf{V}}{\partial x}+V_z\frac{\partial\mathbf{V}}{\partial z}= -\frac{e}{m}\mathbf{E}(x,z,t)+\frac{e}{m}\left[\mathbf{V}\times\int_{-\infty}^{t}\mathrm{rot}\,\mathbf{E}(x,z,t)dt\right]. \qquad (3)$$

Here, $\omega_H=eH/mc$ is the electron gyrofrequency, γ is the effective collision frequency. Eq. 3 should be supplemented by the continuity and wave equations:

$$\partial N_e/\partial t+\mathrm{div}\left(N_e\mathbf{V}\right)=0\,, \qquad (4)$$

$$\frac{\partial^2\mathbf{E}}{\partial t^2}+c^2\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}-4\pi e\frac{\partial}{\partial t}\left(N_e\mathbf{V}\right)=0\,. \qquad (5)$$

We will assume the constancy of the ion density and the quasi-neutrality ($N_i=N_0$=const and $|N_e-N_i|<<N_e,N_i$) and that the probe wave is weaker than the pump ($E_1<<E_2$).

The solution of Eqs. 3-5 can be found using the approach of so-called "reduced" equations which has been used already during the similar investigations [3,5]. Within this approach, only the terms with "resonant" frequencies are retained in Eqs. 3-5. From the resulted system of reduced equations the final expression for the effective refractive index of the probe wave can be derived:

$$\frac{c^2 k_1^2}{\omega_1^2} = 1 - \frac{2(a-b+c)}{2a - b \pm \sqrt{b^2 - 4ac}}. \qquad (6)$$

Here,

$$\begin{aligned} a &= \varepsilon_{xx}\sin^2\Theta_1 + \varepsilon_{zz}\cos^2\Theta_1 + (\varepsilon_{zx} + \varepsilon_{xz})\sin\Theta_1\cos\Theta_1, \\ b &= \varepsilon_{xx}\varepsilon_{zz} + \varepsilon_{yy}\varepsilon_{zz}\cos^2\Theta_1 + (\varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}\varepsilon_{yx})\sin^2\Theta_1 + \\ &\quad (\varepsilon_{yy}\varepsilon_{zx} - \varepsilon_{yx}\varepsilon_{zy} + \varepsilon_{yy}\varepsilon_{xz} - \varepsilon_{xy}\varepsilon_{yz})\sin\Theta_1\cos\Theta_1, \\ c &= \varepsilon_{zz}(\varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}\varepsilon_{yx}). \end{aligned} \qquad (7)$$

The components of dielectric permittivity ε_{jk} have the following form:

$$\varepsilon_{jk} = \varepsilon_{0\,jk} + \frac{A_{jk}(\omega_1, \mathbf{k_1}, \Theta_1)\xi_{EC}}{D(\omega_L, \mathbf{k_L}) - B(\omega_1, \mathbf{k_1}, \Theta_1)\xi_{EC}}. \qquad (8)$$

Here, $\varepsilon_{0\,jk}$ is the "linear" dielectric permittivity of cold magnetized plasma [7] in the coordinate system, shown in Fig. 1, $\xi_{EC}=|V_2|^2/(2\omega_2/k_2)^2$ is the ratio of the squares of the oscillatory and phase velocities for the pump wave, $D(\omega_L,\mathbf{k_L})$ is well-known expression

$$D = \begin{vmatrix} -\frac{c^2 k_{Lz}^2}{\omega_L^2} + \varepsilon_{0\,xx}(\omega_L) & \varepsilon_{0\,xy}(\omega_L) & c^2 k_{Lx} k_{Lz}/\omega_L^2 \\ \varepsilon_{0\,yx}(\omega_L) & -\frac{c^2 k_L^2}{\omega_L^2} + \varepsilon_{0\,yy}(\omega_L) & 0 \\ c^2 k_{Lz} k_{Lx}/\omega_L^2 & 0 & -\frac{c^2 k_{Lx}^2}{\omega_L^2} + \varepsilon_{0\,zz}(\omega_L) \end{vmatrix}, \qquad (9)$$

determining the "linear" dispersive law of the wave with beatwave frequency and wavevector (Re$D(\omega_L,\mathbf{k_L})=0$). Matrix $A(\omega_1,\mathbf{k_1},\Theta_1)$ and scalar $B(\omega_1,\mathbf{k_1},\Theta_1)$ do not depend on ξ_{EC}. The expressions for them are extremely cumbersome and not shown here.

3. Propagation of the probe wave in EIT regime

We will focus on the case, when the frequency of probe wave is close to the upper-hybrid frequency $\omega_{uh}=(\omega_p^2+\omega_H^2)^{1/2}$, because in this frequency range the transversely propagating extraordinary wave has the resonant absorption line and the cutoff region [7].

The quantity ξ_{EC} is a small parameter of the problem: for reasonable pumping intensities (of the order of 10 kW cm^{-2} in microwave frequency range), ξ_{EC} is 10^{-7}. Consequently, the contribution of the second term on the right-hand side of equation (8), which corresponds to the manifestation of EIT, is significant only if $D(\omega_L,\mathbf{k_L})$ is small enough. This implies that the behaviour of the dispersion curves for the probe wave in the EIT region is mainly determined by the "linear" dispersion law of beatwave, Re$D(\omega_L=\omega_1-\omega_2,\ \mathbf{k_L}=\mathbf{k_1}-\mathbf{k_2})=0$. Fig. 2a shows the dispersive curves of probe wave at the upper-hybrid resonance. As it can be seen, their behaviour is indeed defined by the beatwave dispersive law.

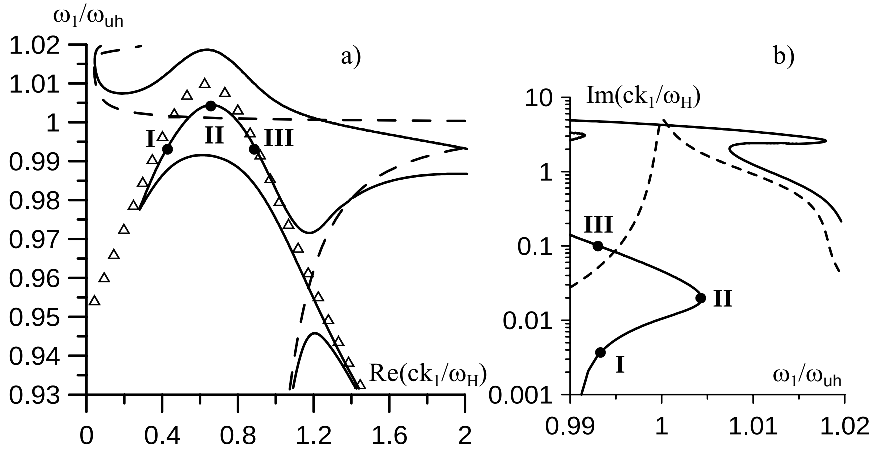

Fig. 2. Dispersion (a) and dissipation (b) of the probe wave in EIT regime. $\xi_{EC}=4.5\cdot10^{-2}$ (solid line), $\xi_{EC}=0$ (dashed line); Re$D=0$ (Δ); $\omega_p/\omega_H=0.2$, $\gamma/\omega_H=5.0\cdot10^{-4}$, $\omega_2/\omega_H=0.83$, $\Theta_2=45°$, $\Theta_1=90°$.

For the longitudinal propagation the group velocity of the probe wave has only the component $V_k=\partial\omega_1/\partial k_1$, directed along the wavevector $\mathbf{k_1}$. In our case the transversal component $V_\perp=-(1/k_1)\partial\omega_1/\partial\Theta_1$ [8] is also present and the total value of group velocity is determined by the expression $(V_k^2+V_\perp^2)^{1/2}$.

Fig. 2b demonstrates the profile of probe wave absorption. It follows from Fig. 2b and from calculations for the expression of refractive index, that there is the region on the dispersive curve (region I-III, see Fig. 2a), for which the strong group velocity slowing-down and the significant reduction of dissipation are simultaneously observed for the probe wave. These are two main consequences of EIT regime. Also it is important, that the effect of EIT is observed in the range of parameters, corresponding to the effective excitation of plasma oscillations at the beatwave frequency:

$$\begin{aligned} \omega_1 &= \omega_2 + \omega_p + O(\xi_{EC}), \\ \mathrm{Re}k_1 &= k_2\sin\Theta_2/\sin\Theta_1 + O(\xi_{EC}). \end{aligned} \qquad (10)$$

This feature of the effect is in the agreement with the results, obtained in works [1,3]: the EIT is observed only when the collective degrees of freedom are effectively excited in the plasma.

The table below presents the dependence of the basic propagation characteristics of the probe wave on the pump intensity I_2. Here, $\gamma/\omega_H=1\cdot10^{-8}$, which corresponds to the Coulomb collisions at the temperature 1 keV, other parameters are the same as those in Fig. 2. In this table $\Delta\omega$ is the transparency bandwidth at level $\mathrm{Im}k_1/\mathrm{Re}k_1$=0.1, $L=(2\mathrm{Im}k_1)^{-1}$ is the absorption length at point II (see Fig. 2b) and V_{gr} is the minimal group velocity. As it can be seen, these parameters increase when pump intensity grows and, moreover, their dependence on I_2 is the same as for the longitudinal propagation [5]:

$$\Delta\omega \sim \sqrt{I_2}\,,\ L \sim I_2\,,\ V_{gr} \sim I_2 \tag{11}$$

I_2, kW cm^{-2}	$\Delta\omega/\omega_{uh}$	L, cm	V_{gr}/c
10	$5\cdot10^{-3}$	20	$6\cdot10^{-7}$
3	$3\cdot10^{-3}$	5	$2\cdot10^{-7}$
1	$1.5\cdot10^{-3}$	1.5	$6\cdot10^{-8}$
0	—	$2.5\cdot10^{-4}$	—

4. Propagation in smoothly inhomogeneous plasma

It is also of significant interest to consider the effect of EIT in inhomogeneous medium. In this case the possible application of EIT in diagnostics can be the transportation of the "trapped" probe wave from the region of overdense plasma (cutoff region) to the vacuum.

Let us consider the dependence of probe wave refractive index on the plasma density. As it can be seen from Fig. 3a, for the finite pump intensity there are two branches AB and CD of dispersive curve ck_1/ω_1 (ω_p), "connecting" the cutoff region (point A) and vacuum (point D) and having the small dissipation ($\mathrm{Im}k_1 \ll \mathrm{Re}k_1$). Strictly speaking, there is no the continuous curve, connecting the regions of cutoff and vacuum. But for the moderate pump intensity (which should, however, exceed the threshold value, determined by the dissipation [1,3]) the gap between the dispersive branches (points B and C) is small and there is the possibility of "transition" (mode conversion) of the wave from one branch to another. The conversion efficiency can be estimated using the well-known technique [8]. These estimations show, that for the pump intensity of 10 kW cm^{-2} and for the plasma inhomogeneity scale of 1 meter the efficiency of more than 90%.

For the parameters, corresponding the Fig. 3a, only the propagation in one direction (from cut-off region to the vacuum) is possible, because the angle between group velocity vector and density gradient is greater, than 90°. However, the inversed propagation is also possible for the negative value of k_{2x} (see Fig. 3b).

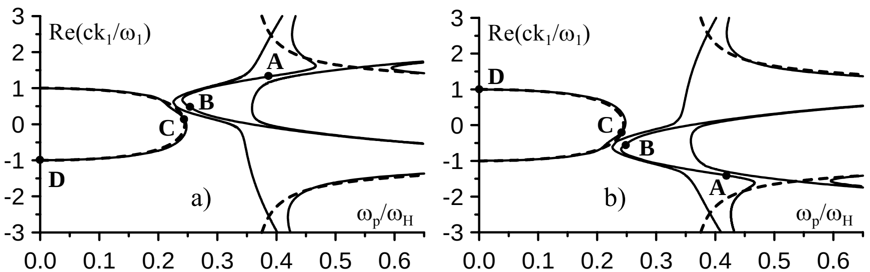

Fig. 3 The refractive index of the probe wave in inhomogeneous plasma. ω_1/ω_H=1.06, $\Theta_2=\pi/4$ (a), $\Theta_2=-\pi/4$ (b), all other parameters are the same as those in Fig. 2.

The interesting feature of EIT regime for quasi-transversal propagation is that, unlike the "linear" case, the behaviour of the group velocity in EIT regime corresponds to the anomalous dispersion of refractive index. This happens due to the dependence of dielectric permittivity (8) on wavevector $\mathbf{k_1}$, "induced" by the pump wave.

Therefore, in EIT regime the probe wave can propagate in smoothly inhomogeneous plasma from the cutoff region to the vacuum (Fig. 3a) and in backward direction: from the vacuum to the overdense plasma (Fig. 3b).

5. Conclusion

We have shown, that the effect of EIT in plasma also exists for the transverse propagation of the radiation relatively to the external magnetic field. There are high group velocity deceleration and the formation of the transparency window within the zone of resonant absorption of the probe wave. This effect is observed in the parameter space, corresponding to the effective excitation of quasi-electrostatic oscillations at the frequency of beating between probe and pumping waves. We also predicted the possibility of probe wave propagation in EIT regime in smoothly inhomogeneous plasma from the region of opaque plasma to the vacuum.

In principle, this effect can be applied for the diagnostics in controlled nuclear fusion for the transportation of the "trapped" probe wave (e.g. spontaneous plasma emission) from the opaque plasma to the vacuum.

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