Ray-Tracing Calculations of Electron Cyclotron Wave Propagation Through Resonance Regions

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Propagation of EC waves in dissipative plasma has been calculated for a range of conditions representing DIII-D ECH experiments using the GENRAY ray-tracing code. The usual wave power balance and the dispersion function are derived assuming a small anti-Hermitian part of the dielectric tensor. However near EC resonance the anti-Hermitian part is comparable with the Hermitian part. In this case the dispersion function has been proposed by E. Westerhof and M. D. Tokman as the real part of the eigenvalue of the dispersion tensor corresponding to the wave mode. In this work we compared the EC ray trajectories for X and O modes calculated using the cold plasma tensor, the Hermitian part of the relativistic dispersion tensor and the Westerhof-Tokman dispersion function for typical DIII-D and RTR conditions.

Wave power flux

The standard approach for the calculation of the electromagnetic wave trajectories by geometric optics ray-tracing techniques uses the Hermitian dielectric tensor $\boldsymbol{\varepsilon}^{H}$. The anti-Hermitian part of dielectric tensor $\boldsymbol{\varepsilon}^{aH}$ near the electron cyclotron layer has the same order of magnitude as the Hermitian part. Under these conditions the group velocity used in the ray-tracing equations can give an incorrect wave power flux. The Pointing theorem in a dissipative medium, proposed by Piliya and Fedorov [1] for the case of non-vanishing anti-Hermitian dielectric tensor suggests

$$\operatorname{div}\mathbf{S}_H + Q + q = 0 \qquad (1)$$

Here $\mathbf{S}_H$ is the standard wave power flux $\mathbf{S}_H = \frac{c^2}{16\pi\omega} E_p^* E_m \frac{\partial D_{pm}^H}{\partial \mathbf{k}}$, $\mathbf{E}$ is the complex amplitude of the wave electric field polarization, D_{pm} is the dispersion tensor $D_{pm} = \delta_{pm}k^2 - k_p k_m - \frac{\omega^2}{c^2}\varepsilon_{pm}(\omega,\mathbf{k})$, Q is the standard term of the dielectric energy flux in a dispersive media $Q = \frac{ic^2}{8\pi\omega} D_{pm}^{aH} E_p^* E_m$, q is the additional term of power looses in a dissipative media with a non-negligible anti-Hermitian part of the dielectric tensor $q = \frac{c^2}{16\pi\omega}\frac{\partial D_{pm}^{aH}}{\partial \mathbf{k}}\left(E_p^*\frac{\partial E_m}{\partial \mathbf{r}} - \frac{\partial E_p^*}{\partial \mathbf{r}}E_m\right)$

It was proposed in [2,3] to transform the Pointing equation to the divergence form

$$\operatorname{div}\mathbf{S}_M + \tilde{Q} = 0 \qquad (2)$$

Here $\mathbf{S}_M$ is the corrected dielectric wave energy flux $\mathbf{S}_M = \frac{c^2}{16\pi\omega} I \frac{\partial(D_{pm}^H e_p^* e_m)}{\partial \mathbf{k}}$,

$\tilde{Q}$ is the corrected source term $\tilde{Q} = \frac{ic^2}{8\pi\omega} I (D_{pm}^{aH} e_p^* e_m)_{\mathbf{k}=\mathbf{k}_0+\nabla\phi}$

$I = |\mathbf{E}|^2$, $\mathbf{e}$ is the unit wave polarization vector, which is an eigenvector of the dispersion tensor $D_{pm}e_m = \lambda^{\text{mode}} e_p$. The determinant of the dispersion tensor is equal to the product of eigenvalues $D = \lambda^1\lambda^2\lambda^3$. The real part of the eigenvalue is $\operatorname{Re}\lambda^{\text{mode}} = e_p^* D_{pm}^H e_m$. The wave energy flux can be rewritten using the real part of the eigenvalue $\mathbf{S}_M = \frac{c^2}{8\pi\omega} I \frac{\partial(\operatorname{Re}\lambda)}{\partial \mathbf{k}}$. The equation

$$\lambda(\mathbf{k}) = 0 \qquad (3)$$

is used as the dispersion equation for the given wave mode.

Ray-Tracing equations

The wave energy flux $\mathbf{S}_M$ and the derivatives $\partial(\operatorname{Re}\lambda)/\partial\mathbf{k}$ have the same direction. It means that the real part of the dispersion tensor eigenvalue

$D = \operatorname{Re}\lambda$ can be used as the dispersion function in the ray-tracing equations. In this case the ray-tracing equations have the form

$$\frac{\partial \mathbf{r}}{\partial l} = -\frac{\partial(\operatorname{Re}\lambda)}{p\partial \mathbf{N}}, \quad \frac{\partial \mathbf{N}}{\partial l} = \frac{\partial(\operatorname{Re}\lambda)}{p\partial \mathbf{r}}, \qquad (4)$$

$$p = \left|\frac{\partial(\operatorname{Re}\lambda)}{\partial \mathbf{N}}\right|$$

Here $\mathbf{N} = c\mathbf{k}/\omega$ is the refractive index, l is a length along the ray. The group velocity is not used in this form of the ray-tracing equations.

GENRAY ray-tracing code

GENRAY ray-tracing code [4] calculates the electromagnetic waves propagation in toroidal plasmas for the several dispersion tensors: cold plasma, hot non-relativistic plasma, relativistic electron plasma with the Mazzucato approximation of the dielectric tensor [4]. We have added to GENRAY the corrected dispersion function (4) proposed in [2,3] for ECH waves calculations. The dispersion tensor eigenvalues are complex roots of the cubic equation $a\lambda^3 + b\lambda^2 + c\lambda + d = 0$. The coefficients of this equation are complex numbers. They are functions of the dispersion tensor elements D_{pm}. We used the Cardan formulas for finding the roots of the cubic equation.

EC O-mode and X-mode in RTP

We have made ray-tracing calculations with GENRAY using the dispersion function $D = \operatorname{Re}\lambda$ for non-Hermitian dielectric tensor for EC waves in RTR and for the parameters used in [6] in order to be able to compare results with Westerhof [6]. The rays were launched at different poloidal angles for 60 GHz O-mode and 110 GHZ 2-nd harmonics X-mode. The results are presented in Fig.1 and Fig.2. The dispersion function $D = \operatorname{Re}\lambda$ gave the deflection of the rays from the straight lines obtained in the cold plasma fox O-mode for the poloidal launch angles ~ $90^0 \pm 20^0$ and for X-mode for the poloidal launch angles ~ $90^0 \pm 30^0$. Similar results were presented by Westerhof [6].

ECR X-mode second harmonic in DIII-D

Ray trajectories at 110 GHz for the second harmonic X-mode were calculated for the typical DIII-D conditions. In Fig. 3 one can see the results of GENRAY calculations using the dispersion function $D = \operatorname{Re}\lambda$ for non-Hermitian dielectric tensor. In this case the hot plasma effect gives a significant deflection of trajectories for the poloidal launch close to the vertical line.

Conclusion

In the GENRAY code the Westerhof-Tokman dispersion function for the case when anti-Hermitian part of dielectric tensor has the same order of magnitude as the Hermitian part, was implemented.

Rays presented in EC-10 [6] by Westerhof for O1 and X2 modes in the RTP tokamak showed good agreement with GENRAY rays.

Reflection was obtained for rays grazing the O1 and X2 resonances, whereas for cold plasma the rays were close to straight lines.

Using the Mazzucato-Fidone-Granata relativistic dielectric tensor, the Westerhof-Tokman dispersion function gave modified rays compared to the near-Hermitian dispersion approach. This was for RTP (O1 and X2) and DIII-D (X2) cases.

Ray-tracing calculations show the largest effects of the warm relativistic plasma dispersion near the cyclotron resonances for near vertical launch of the waves.

Acknowledgements

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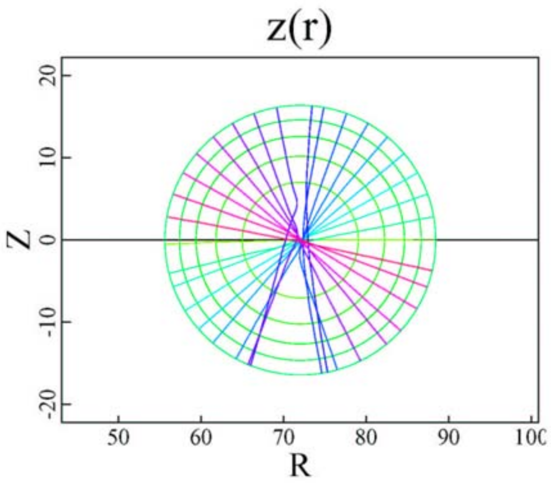

Fig.1. O-mode in RTR for non-Hermitian relativistic dielectric tensor

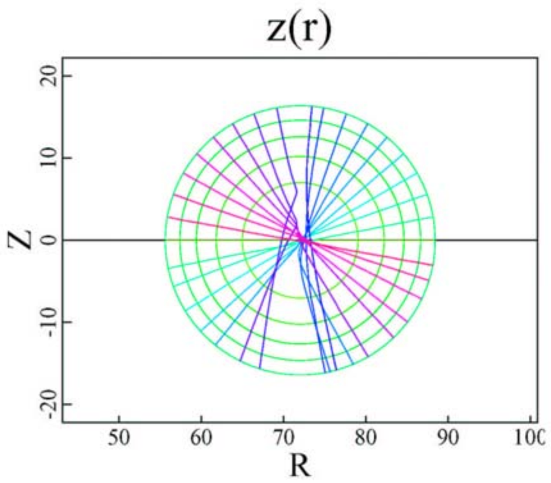

Fig. 2 X-mode second harmonics in RTR for non-Hermitian relativistic dielectric tensor using the dispersion function $D = \mathrm{Re}\,\lambda$

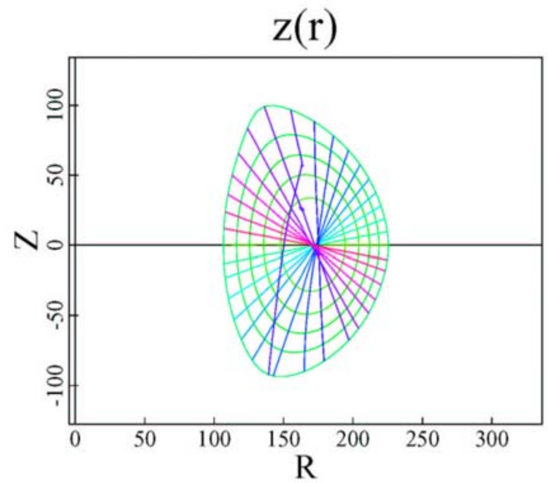

Fig.3 EC 110 GHz second harmonics X-mode in DIII-D non-Hermitian relativistic dielectric tensor using the dispersion function $D = \mathrm{Re}\,\lambda$