On absence of relativistic damping of electron Bernstein waves in tokamak plasmas

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Outline

- "Traditional" scheme of EBW propagation from low field side.
- Waves launched far and close to mid-plane.
- Rays calculations. EBWs damping
- Full wave analysis. Quantum mechanics analogy.
- Summary

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“Traditional” scheme of EBW propagation from low field side in a ST

- As far as *ECR* heating is concerned, main feature of STs is high plasma density at comparatively low magnetic field. Relevant dimensionless parameter $\omega_{pe}^2/\omega_{ce}^2$ in the plasma center is of order unity for "conventional" tokamaks and equal to 50-100 in ST. As a result, plasma interior is not accessible for 1st and 2nd electro-magnetic wave harmonics. Only accessible to (but not optically thick for) higher harmonics.

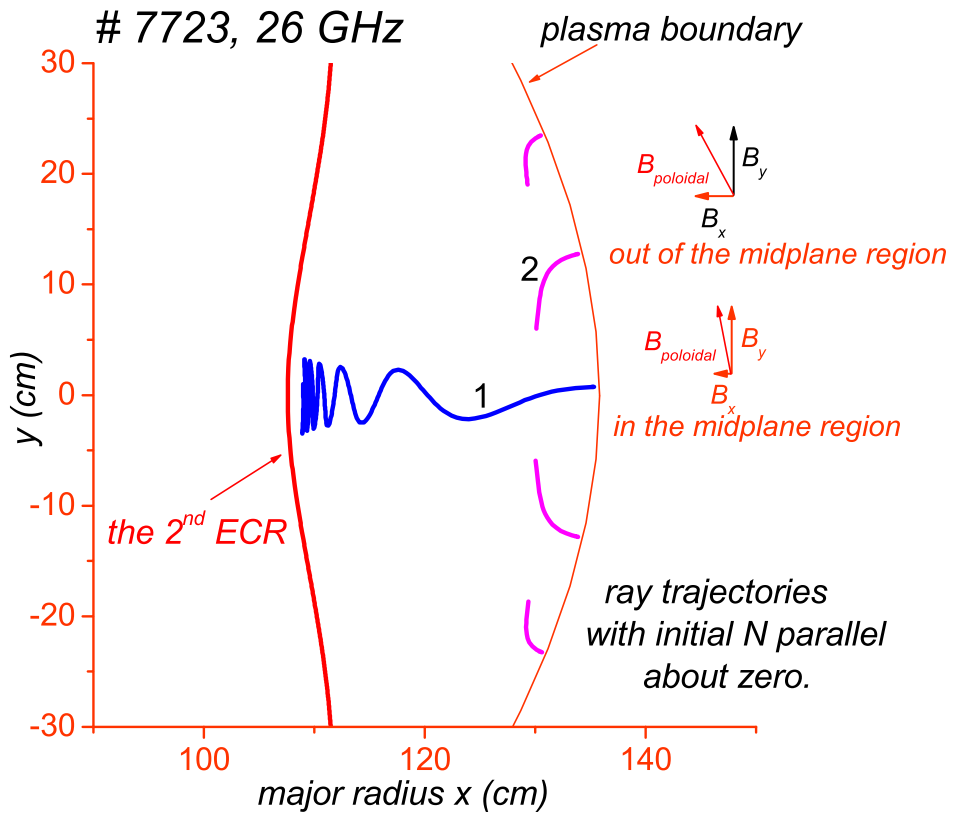
- Plasma interior is accessible and optically thick for *EBW* produced via linear conversion of incident electromagnetic waves in the *UHR* region. Conversion process may include $O \rightarrow X \rightarrow B$ transformation, direct tunneling of combination of both.

EBWs launched far and close to mid-plane

EBWs are born in UHR vicinity due to linear conversion with $|n_{||}| \leq 1$

1) *far from mid-plane EBWs are damped quickly* $n_{||} \simeq \alpha n_x \geq 1$

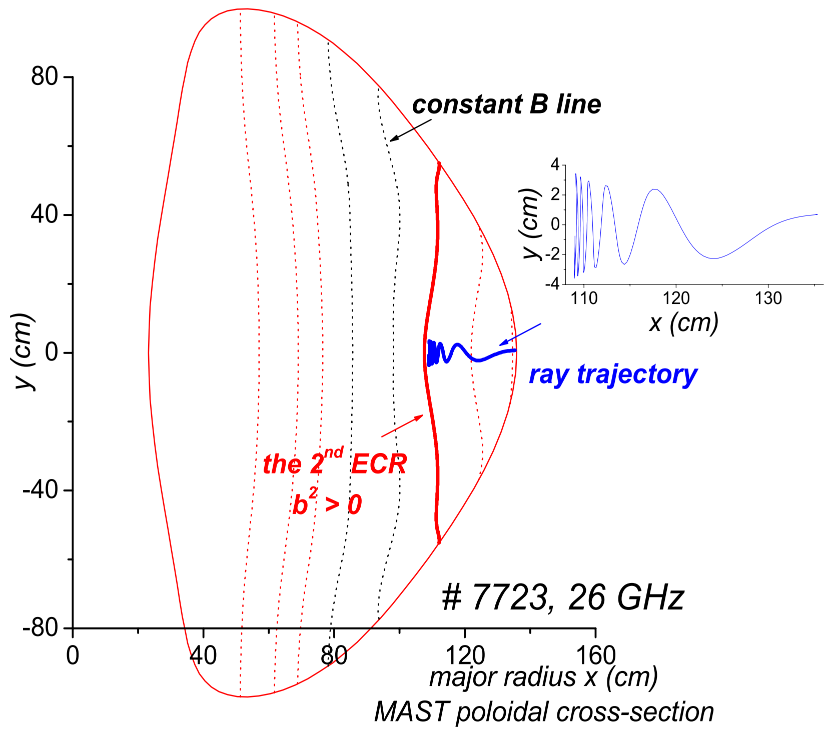
2) *in mid-plane EBWs are capable of propagation far inward plasma*

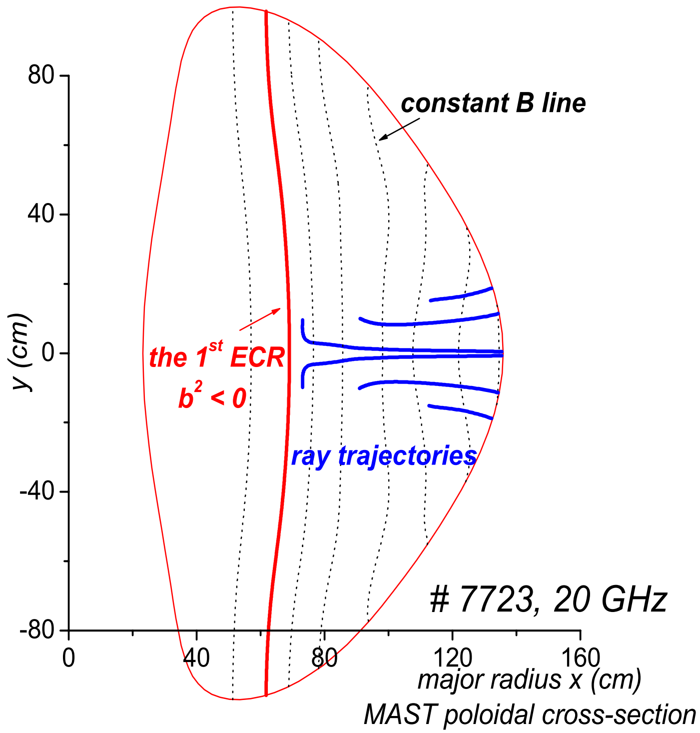


MAST poloidal cross-section

EBWs launched close to mid-plane

!!! EBWs close to mid-plane propagate *far* inward plasma





!!! In the case of concave magnetic field lines strange oscillatory rays behavior occur

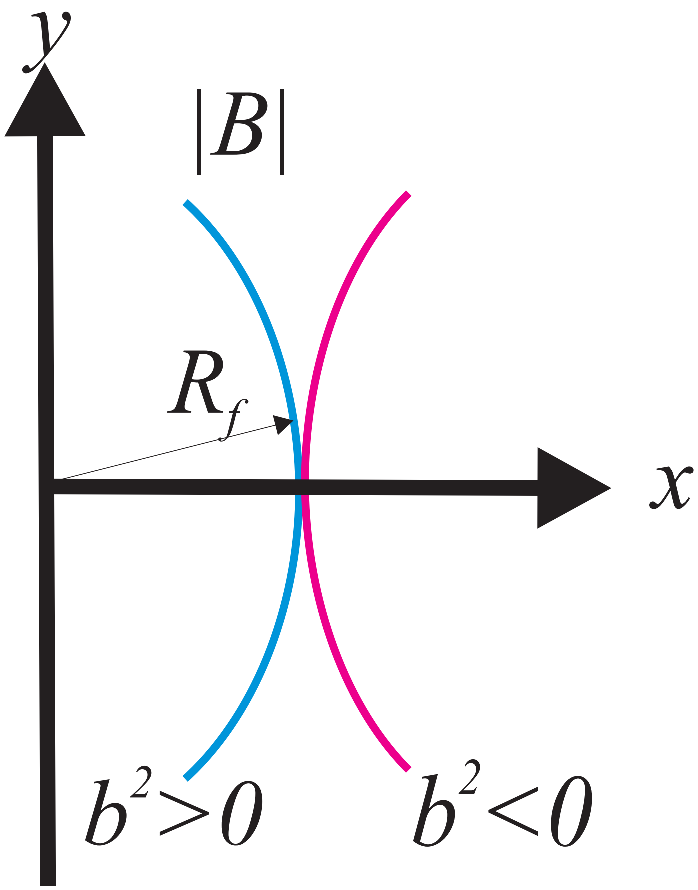
Model and assumptions

Assumptions

1) $n_x \gg 1 \Rightarrow \varepsilon = 0$

2) $\dfrac{n_{\|}\beta}{\omega - m\omega_{ce}} \ll 1$

3) $\omega_{ce}(x,y) \approx \omega_{ce}(x,0)\left[1 - \dfrac{y^2}{b^2} + \ldots\right]$



Wave-vector's parallel projection

$$k_{\|} = \frac{\vec{k}\vec{B}}{B} = \frac{B_p}{B}\left(k_y + \frac{B_T}{B_P}k_z - \frac{y}{R_F}k_x\right)$$

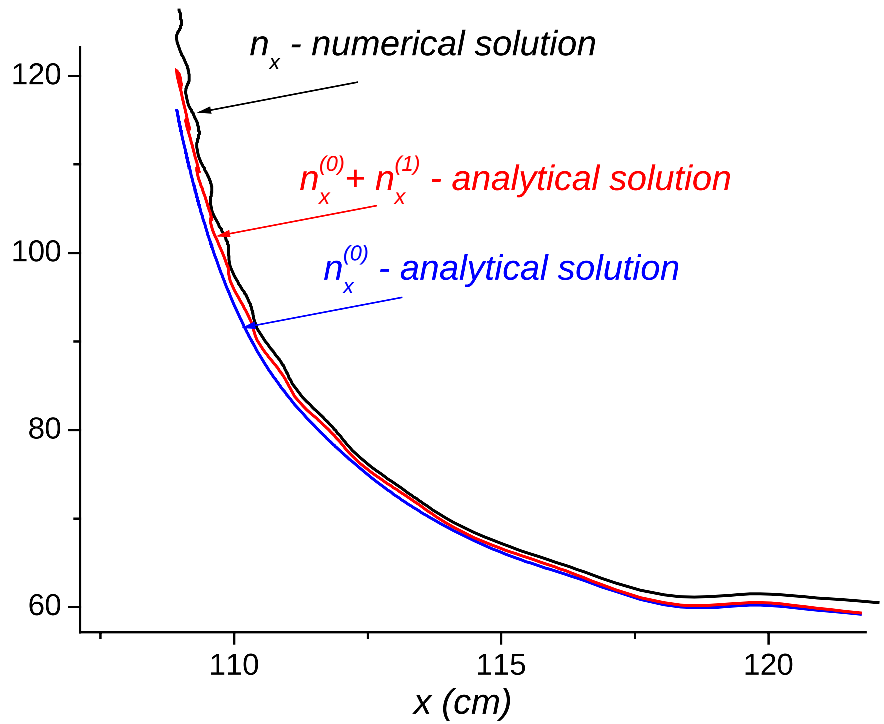
EBWs dispersion relation

$$\varepsilon = \varepsilon_0 + \varepsilon_1 = 0$$

$$\varepsilon_0 = 1 + \frac{2\upsilon}{n_x^2 \beta^2}\left(1 - \frac{1}{\sqrt{\pi} m n_x \beta \Delta}\right)$$

$$\varepsilon_1 = -\frac{2\upsilon}{\sqrt{\pi}(n_x\beta)^3 m\Delta}\left(\frac{y^2}{\Delta b^2} - \frac{(\beta n_\parallel)^2}{\Delta^2}\right)$$

$$\Delta \equiv \frac{q(x,0) - m}{q(x,0)} = \frac{x - x_{ECR}}{R}$$



$$n_x \simeq n_x^{(0)}(x) + n_x^{(1)}\left(n_x^{(0)}(x); y, n_\parallel\right)$$

$$\varepsilon_0 = 0 \Rightarrow n_x^{(0)}(x); \quad \varepsilon_1 \Rightarrow n_x^{(1)}\left(n_x^{(0)}(x); y, n_\parallel\right) = -\frac{\varepsilon_1\left(n_x^{(0)}(x); y, n_\parallel\right)}{\partial\varepsilon_0 / \partial n_x\big|_{n_x = n_x^{(0)}}}$$

Rays equations

$$\frac{dy}{d\tau} = -\frac{\partial K}{\partial n_y}, \quad \frac{dn_y}{d\tau} = \frac{\partial K}{\partial y}$$

$$K = \frac{\delta^{1/2} y^2}{b^2} + \frac{n_{\|}^2}{2\delta^{1/2}}, \quad \delta(\tau) = \frac{\Delta(\tau)}{\beta^2}$$

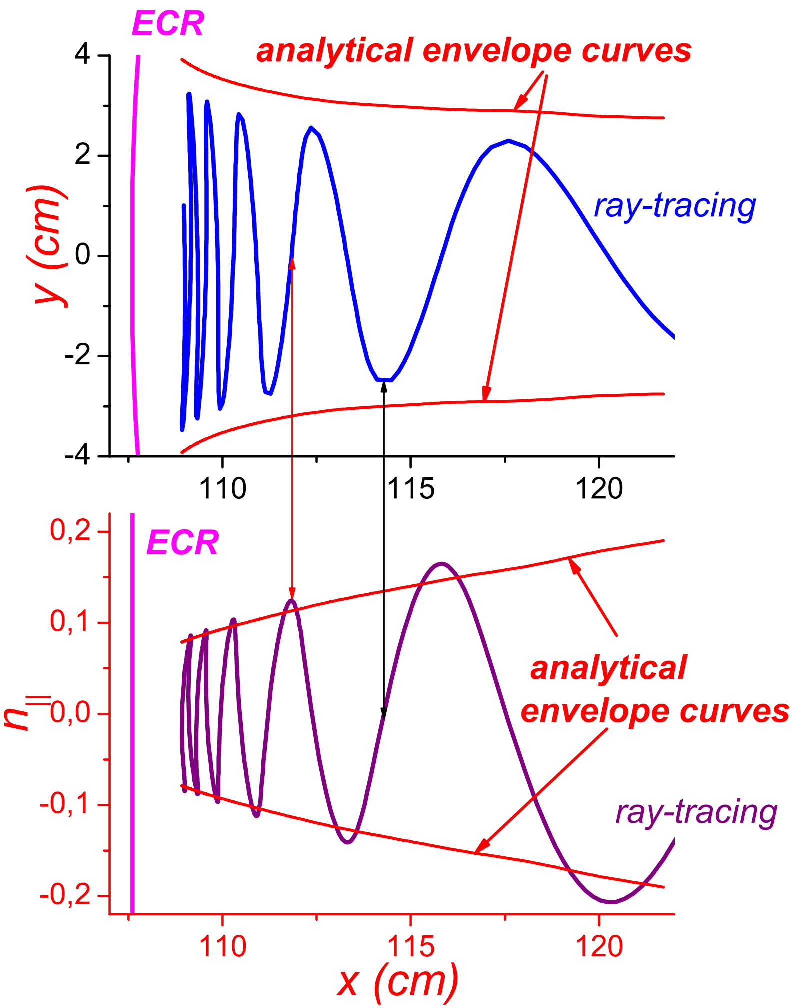
Adiabatic condition

$$\Omega = \frac{B_p}{\sqrt{2}Bb}; \Omega\tau \sim \frac{L}{b\beta n_{\|}} \gg 1; \; I = \frac{K}{\Omega} = const$$

Solutions

$$y = Y \sin\left(\int^{\tau} \Omega(\tau')d\tau' + \alpha\right)$$

$$n_{\|} = N \cos\left(\int^{\tau} \Omega(\tau')d\tau' + \alpha\right)$$



EBWs damping

$$P(x) \sim \exp(-2\Gamma)$$

$$\Gamma = \int^{x} \frac{\varepsilon''(n_{x0}, n_{\|}, x')}{\partial\varepsilon_0 / \partial n_x \big|_{n_x = n_x^{(0)}}} dx'$$

$$\Gamma(\infty) \sim \frac{(2\pi\upsilon)^{1/3} n_{\|}^{2/3}}{3\beta^{1/3} m^{1/3}} \frac{\omega l}{c} \gg 1$$

Wave equation's solution

One can draw a conclusion that in the case of concave ECR surface there is a sort of the "potential well" in the poloidal direction, slowly varying with x. The EBWs must propagate in this plasma wave-guide as a discrete set of eigenmodes whose damping might be different from one predicted by the ray theory.

$$\nabla^2\varphi + 4\pi\rho(\varphi) = 0$$

$$\varphi(x,y) = \frac{e^{i\left(\int^x k_{x0}(x')dx' + \frac{B_T}{B_p}k_z y + \frac{k_{x0}y^2}{2R_F}\right)}}{\sqrt{y_0 n_{x0}^2 \frac{\partial\varepsilon_0}{\partial n_{x0}}}} \sum_{n=-\infty}^{\infty} C_n F_n(y,\tau) e^{i\int_0^\tau K_n(\tau')d\tau'}$$

$$F_n'' + (I_n - \xi^2)F_n = 0$$

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Quantum mechanic analogy

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{kx^2}{2} \quad \Rightarrow \quad \hat{p} = i\hbar\frac{\partial}{\partial x} \qquad \min E = \frac{\hbar\omega}{2}$$

$$K = \frac{\delta^{1/2} y^2}{b^2} + \frac{n_{\parallel}^2}{2\delta^{1/2}} \quad \Rightarrow \quad n_{\parallel} \to -i\frac{B_p}{B}\frac{\partial}{\partial y} \qquad \min\left(n_{\parallel}^2\right) \simeq \frac{B_p}{B}\frac{c}{\omega_{ce} b}$$

$$\min\left(n_{\parallel}^2\right) \gg \beta^2$$

To describe EBWs damping the relativistic dispersion function is not necessary

Conclusions

1.The *EBWs* propagating close to the tokamak mid-plane and capable of penetrating deep into the plasma are considered.
2. Ray behaviour in the mid-plane region depends on the *ECR* surface shape. In the case of concave *ECR* surface the rays oscillate around the mid-plane while for the convex *ECR* surface their deviation from the mid-plane growths exponentially.
3. In the case of oscillating rays the ray equation has an additional (adiabatic) integral of motion. The presence of this integral explains basic features of ray trajectories.
4. Conception of rays is incorrect for small $n_{||}$ case. The relevant part of the EBW beam is described by ***low-order*** eigenmodes of the wave equation found in the present work.
5.There is effectively a minimal value of $n_{||}$. In case when its value is sufficiently large compared to β, relativistic effects in wave propagation and damping can be ignored.