Open questions in electron cyclotron wave theory

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This is not a Review!

"theory of electron cyclotron waves is well established"

Yet, gaps in our knowledge remain

- Linear theory of wave propagation and absorption
  - Homogeneous plasma
  - Inhomogeneous plasma
- Quasi-linear theory
- Nonlinear theory
  - Nonlinear absorption
  - Electromagnetically induced transparency
- Et cetera …

… but rather an overview of my ignorance

Linear theory of wave propagation and absorption

- What do we really know about the wave power balance?
  - The text book answer

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} + Q = 0,$$

WRONG!

$$\begin{aligned}
W &= \frac{1}{16\pi\omega_0}\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\mathrm{H}}_{ij})|_{\vec{r}=\vec{r}_0} E_i^* E_j \\
&= -\frac{c^2}{16\pi\omega_0}\frac{\partial}{\partial\omega}(D^{\mathrm{H}}_{ij})|_{\vec{r}=\vec{r}_0} E_i^* E_j, \\
\mathbf{S} &= \frac{c^2}{16\pi\omega_0}\frac{\partial}{\partial\mathbf{k}}(D^{\mathrm{H}}_{ij})|_{\vec{r}=\vec{r}_0} E_i^* E_j, \\
Q &= \frac{i\omega_0}{8\pi}(\varepsilon^{\mathrm{aH}}_{ij})|_{\vec{r}=\vec{r}_0} E_i^* E_j = \frac{ic^2}{8\pi\omega_0}(D^{\mathrm{aH}}_{ij})|_{\vec{r}=\vec{r}_0} E_i^* E_j,
\end{aligned}$$

Tokman et al. NF 2003

... the correct answer is:

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} + Q = 0,$$

$$\begin{aligned}
W &= |A|^2 \frac{1}{16\pi\omega_0}\frac{\partial}{\partial\omega}(\omega^2 \varepsilon^H_{ij} e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0} = -|A|^2 \frac{c^2}{16\pi\omega_0}\frac{\partial}{\partial\omega}(D^H_{ij} e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0}, \\
\mathbf{S} &= |A|^2 \frac{c^2}{16\pi\omega_0}\frac{\partial}{\partial\mathbf{k}}(D^H_{ij} e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0}, \\
Q &= |A|^2 \frac{\mathrm{i}c^2}{8\pi\omega_0}\left(D^{aH}_{ij} e_i^* e_j\right)\Big|_{\vec{\Gamma}=\vec{\Gamma}_0+(\partial\phi/\partial\vec{\xi})}.
\end{aligned}$$

- Deceptively similar, but **crucially different when ε^{aH} is non-negligible** (as near cyclotron resonance)

Tokman et al. NF 2003

... the correct answer is:

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} + Q = 0,$$

$$W = |A|^2 \frac{1}{16\pi\omega_0} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon_{ij}^H e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0} = -|A|^2 \frac{c^2}{16\pi\omega_0} \frac{\partial}{\partial \omega} (D_{ij}^H e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0},$$

$$\mathbf{S} = |A|^2 \frac{c^2}{16\pi\omega_0} \frac{\partial}{\partial \mathbf{k}} (D_{ij}^H e_i^* e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0},$$

$$Q = |A|^2 \frac{\mathrm{i}c^2}{8\pi\omega_0} \left(D_{ij}^{aH} e_i^* e_j \right)\Big|_{\vec{\Gamma}=\vec{\Gamma}_0+(\partial\phi/\partial\vec{\xi})}.$$

- But still *W* and *S* are not the true wave energy density and wave energy flux
  - Final answer is still open requiring a microscopic model of the medium

Tokman et al. NF 2003

… the correct answer is:

$$\frac{\partial W}{\partial t} + \nabla\cdot\mathbf{S} + Q = 0,$$

$$W = |A|^2 \frac{1}{16\pi\omega_0}\frac{\partial}{\partial\omega}(\omega^2\varepsilon^H_{ij}e^*_i e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0} = -|A|^2\frac{c^2}{16\pi\omega_0}\frac{\partial}{\partial\omega}(D^H_{ij}e^*_i e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0},$$

$$\mathbf{S} = |A|^2\frac{c^2}{16\pi\omega_0}\frac{\partial}{\partial\mathbf{k}}(D^H_{ij}e^*_i e_j)|_{\vec{\Gamma}=\vec{\Gamma}_0},$$

$$Q = |A|^2\frac{\mathrm{i}c^2}{8\pi\omega_0}\left(D^{aH}_{ij}e^*_i e_j\right)\Big|_{\vec{\Gamma}=\vec{\Gamma}_0+(\partial\phi/\partial\vec{\xi})}.$$

- … better viewed as equation for evolution of wave intensity

$$-\frac{\partial\,\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}}{\partial\omega}\bigg|_{\vec{\Gamma}=\vec{\Gamma}_0}\frac{\partial|A|^2}{\partial t} + \frac{\partial\,\mathcal{R}\mathrm{e}\,\lambda^{\mathrm{mode}}}{\partial\mathbf{k}}\bigg|_{\vec{\Gamma}=\vec{\Gamma}_0}\cdot\nabla|A|^2 = 2\,\mathcal{I}\mathrm{m}\,\lambda^{\mathrm{mode}}|_{\vec{\Gamma}=\vec{\Gamma}_0+(\partial\phi/\partial\vec{\xi})}\,|A|^2.$$

Wave propagation in inhomogeneous, warm plasma

- Method of choice: geometric optics ray-tracing
  - Wave length small compared to scale of inhomogeneity: λ << R, a
- Proper ray Hamiltonian for warm plasma dispersion:

$$\frac{d\mathbf{r}}{d\tau} = \frac{\partial \mathcal{R}e\, \lambda^{\mathrm{mode}}}{\partial \mathbf{k}} \qquad \frac{d\mathbf{k}}{d\tau} = -\frac{\partial \mathcal{R}e\, \lambda^{\mathrm{mode}}}{\partial \mathbf{r}}$$

- Possibly **enormous** difference between trajectories from cold and warm plasma dispersion!

Tracing focussed beams

- Ray-tracing breaks down near focus
- Better technique: beam tracing (example TORBEAM)
  - Gaussian beams
  - Uses ordering:

$$\lambda/L = \mathcal{O}(\kappa) \ll w/L = \mathcal{O}(\kappa)^{1/2} \ll 1 \qquad L = a, R$$

  - Solves trajectory of central ray (as usual) plus ODE's for beam width and phase front curvature
  - Absorption assumed constant over beam cross section and based solely on properties of central ray

Improving beam tracing I

- Resonance broadening

$$\delta(\gamma - n\omega_{ce}/\omega - N_{\|}p_{\|}/m_e c) \rightarrow \sqrt{\frac{\pi}{2\Delta Q}} \exp\left[-\frac{(\gamma - n\omega_{ce}/\omega - N_{\|}p_{\|}/m_e c)^2}{2\Delta Q}\right]$$

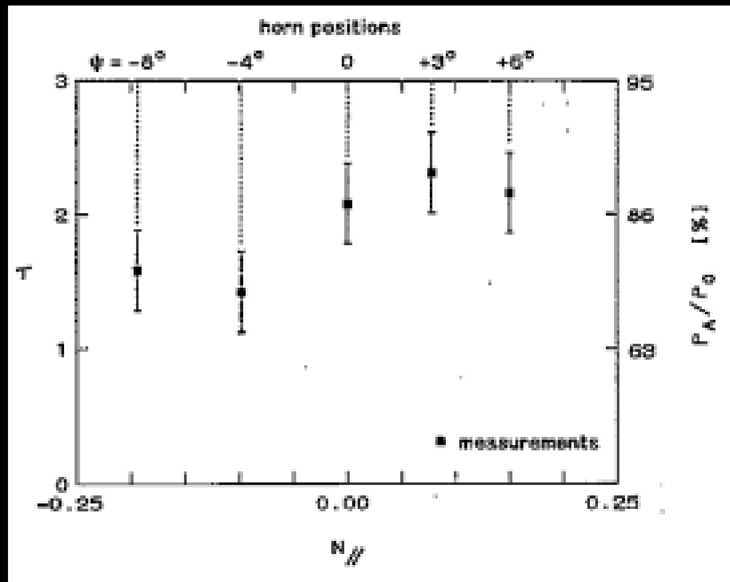
$$\Delta Q = \left(\frac{\gamma v_{\|}}{\omega w}\right)^2 + \left(\frac{\gamma \Delta N_{\|} v_{\|}}{2c}\right)^2$$

- Broadens deposition profile for strongly focussed beams

Improving beam tracing II

- Include effects of warm plasma dispersion
- Inhomogeneity of absorption across the beam
- Asymmetries in absorption due to asymmetric electron distribution

Example from RTP



- Consequences for beam shape and trajectory?
- In all cases ordering breaks down: w/L is not small
  - Dispersion changes over scale resonance: $L = \Delta R_{res}$

Electron Bernstein waves

- Always involves one or more mode conversions
  - O-X-B
  - Direct X-B tunnelling through cut-off layer
- Open issues

  in addition to those already mentioned for beam tracing
  - Beam tracing across a mode conversion region
    - What happens when focal point is beyond mode conversion?
    - How best to focus an EBW beam?
    - How best to localize the power deposition / current drive?
  - Are relativistic effects important in EBW propagation?

Yes, relativistic effects are important also for EBW!

- Ram et al., EPS 2003 (3 keV, ω_{pe}/ω_{ce}=6, ω/ω_{ce}=1.8):

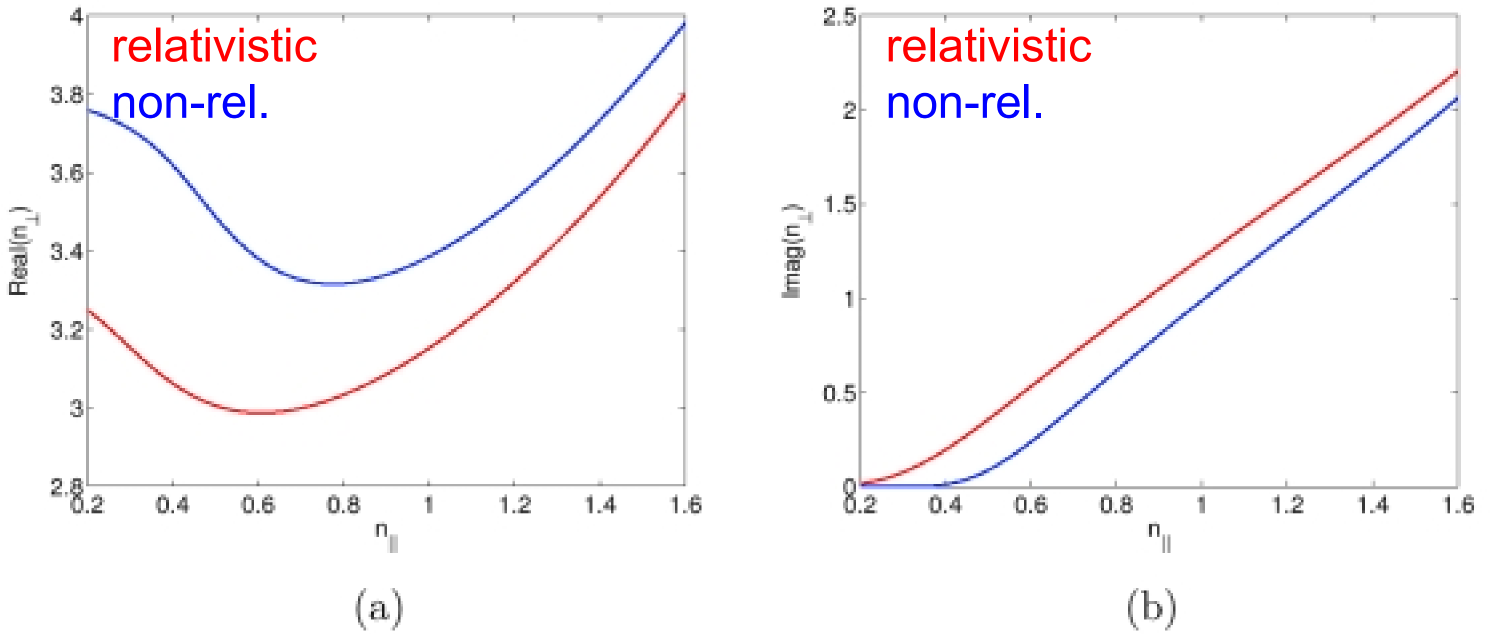


Figure 2: Dispersion characteristics of the EBW as a function of $n_{\|} = ck_{\|}/\omega$: (a) real part of $n_\perp$; (b) imaginary part of $n_\perp$. The parameters are as cited in the text. The red color represents the solution of the dispersion relation in the fully relativistic case, and the blue color is for the non-relativistic case.

Quasi-linear theory

- Describes plasma response consistent with linear wave theory balanced by collisions (and radial transport)
- Our basic tool:
  bounce averaged quasi-linear Fokker-Planck equation

$$p_0^2 \lambda \sin\theta_0 \frac{\partial f_0(r_0,p_0,\theta_0)}{\partial t} = \frac{\partial}{\partial I_{0i}} p_0^2 \lambda \sin\theta_0 \left(\left\langle \frac{\partial I_{0i}}{\partial \mathbf{u}} \cdot \mathbf{D}^{\mathbf{uu}} \cdot \frac{\partial I_{0j}}{\partial \mathbf{u}} \right\rangle \frac{\partial}{\partial I_{0j}} - \left\langle \frac{\partial I_{0i}}{\partial \mathbf{u}} \cdot \mathbf{F}^{\mathbf{u}} \right\rangle \right) f_0(r_0,p_0,\theta_0)$$

  (r_0, p_0, θ_0) are radius, momentum and pitch angle at the low field side (invariants) and u are local velocity space coordinates

- Bounce averaged QLFP contains all neoclassical effects provided drift orbits are used in $\partial r_0/\partial \mathbf{u}$
  – r_0 is taken to be constant in most BAQLFP codes

Neoclassical aspects I

- In particular, terms responsible for bootstrap current
  - i.e. momentum space flux driven by radial gradient

$$\Gamma^i_{\rm neo} = \left\langle \frac{\partial I_{0i}}{\partial \mathbf{u}} \cdot \mathbf{D}^{\rm uu} \cdot \frac{\partial r_0}{\partial \mathbf{u}} \right\rangle \frac{\partial}{\partial r_0} f_0(r_0, p_0, \theta_0) \qquad i = (\theta_0, p_0)$$

  - dominant contribution from trapped passing boundary

$$\Gamma^i_{\rm neo} \approx \Gamma^{\theta_0}_{\rm neo,t/p} = \mp\delta(\theta_0 - \theta_{0,\rm t/p})\mathbf{D}^{\theta_0\theta_0} w_b \frac{\partial}{\partial r_0} f_0(\mathbf{I}_0),$$

  - easily included into existing codes: CQL3D, RELAX

Neoclassical aspects II

- Alternative method for inclusion of neoclassical terms

Decker, Peysson, et al.

  - Expansion of drift kinetic equation
  - Zero order equals BAQLFP (with constant r_0)
  - First order correction in steady state $f_1 = \tilde{f} + g$, with finite drift orbit perturbation $\tilde{f}$ and corresponding plasma response g

$$\left\langle D(g) \right\rangle = -\left\langle D(\tilde{f}) \right\rangle$$

  - where D includes effects from RF and collisions
  - both methods give similar results

- Application: RFCD-bootstrap synergy

Self-consistent current density evolution

- How does the plasma current respond to ECCD?
  - An electric field will be generated to compensate $\partial j/\partial t$
  - ECCD-V_{loop} synergy, so Ohm's law cannot be used

- Use Faraday:

$$V_{\text{loop}} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{S},$$

Nonlinear theory

- Reviews: Cohen et al. RMP 63 (1991) 949
  Litvak et al. PoF B 5 (1993) 4347

- This talk highlights just two issues
  - Nonlinear absorption
  - Electromagnetically induced transparency or EIT

Is nonlinear absorption important?

- Ratio coherent interaction over nonlinear trapping times

$$\varepsilon_{NL} \equiv \frac{\tau_{\rm co}}{\tau_{\rm tr}},$$

$$\tau_{\rm co}(w) = \frac{2w}{v_\parallel}.$$

- Further limit on coherent interaction from inhomogeneity of plasma and beam parameters
  - Magnetic field variation along orbits passing beam
  - Beam phase front curvature R_c (i.e. $\Delta N_\parallel$)

$$\tau_{\rm co}({\rm inhom.}) = \left| v_\parallel \frac{\partial\left(\omega_{ce} - k_\parallel v_\parallel\right)}{\partial s}\right|^{-\frac{1}{2}},$$

$$\tau_{\rm co}(R_c) = \sqrt{\frac{R_c c}{\omega v_\parallel^2}}.$$

Is nonlinear absorption important?

- Ratio coherent interaction over nonlinear trapping times

$$\varepsilon_{NL} \equiv \frac{\tau_{\rm co}}{\tau_{\rm tr}},$$

$$\tau_{\rm co}(w) = \frac{2w}{v_\parallel}.$$

$$\tau_{\rm tr} = \sqrt{\frac{4\pi}{(\partial\gamma/\partial t)_{QL}}}.$$

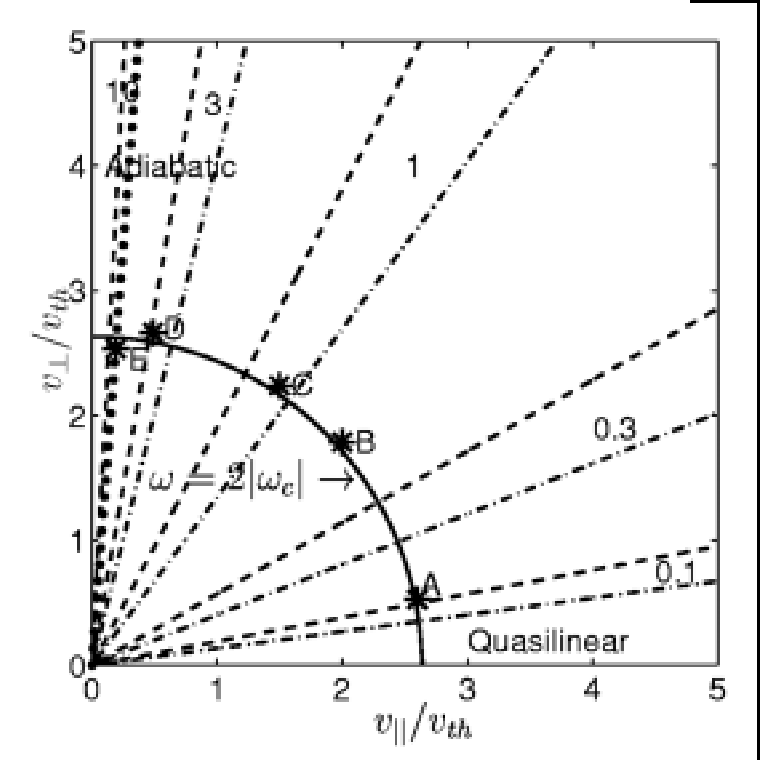
$$\frac{\partial m_e c^2 \gamma}{\partial t} \approx v_\perp \left(\frac{1}{\sqrt{2}} (eE_- J_{n-1} + eE_+ J_{n+1}) + \frac{v_\parallel n \omega_{ce}}{v_\perp \gamma \omega} eE_\parallel J_n \right)$$

YES, fraction of phase space is always (weakly) nonlinear

$$\varepsilon_{\mathrm{NL}}(w) \approx \begin{cases} 0.4 \quad \tan\theta \ \sqrt{\dfrac{w}{\lambda}}\, P_0^{1/4}[MW] & \text{for } n = 2 \text{ X-mode} \\ \\ 0.4 \sqrt{\tan\theta}\ \sqrt{\dfrac{w}{\lambda}}\, P_0^{1/4}[MW] & \text{for } n = 1 \text{ O-mode,} \end{cases}$$

Illustration of ε_{NL} for perpendicularly propagating X-mode at second harmonic resonance

P_{140GHz} = 0.1 and 0.4 MW, w = 2 cm

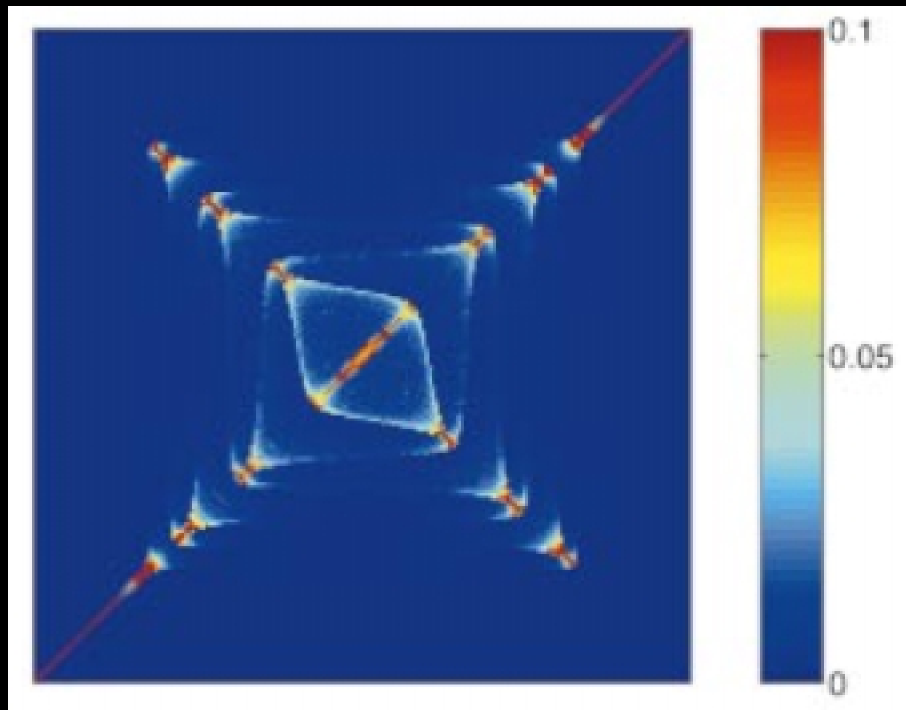


Nonlinear absorption

- Description by means of transition probability for beam crossings (Kamendje et al. PoP 10 (2003) 75)
  - Individual crossings can be assumed uncorrelated
  - Implemented in Monte Carlo code

$P(p_\perp{}^2{}_{before}, p_\perp{}^2{}_{after})$

$\varepsilon_{NL} = 5.8$



How to proceed?

- Add more realism to model
  - Finite phase front curvature
  - important when $R_c\ c/\omega < 9\ w^2$ (Kamendje 2004)
  - Magnetic field inhomogeneity across beam

- Required extensions / Open issues
  - Add parallel acceleration (in particular for oblique 2nd X)
  - Distribution of absorption over the beam and consequences for the beam evolution
    - Nonlinearity varies across the beam
    - Power absorption will vary along particle trajectories
    - Plasma inhomogeneity over beam width

Electromagnetically Induced Transparency (EIT)

Litvak&Tokman PRL2002

- EIT well known quantum mechanical phenomenon in three level system
- Classical (plasma) analog involves three waves

low power probe

$(\omega_{\mathrm{probe}}, k_{\mathrm{probe}})$

high power pump

$(\omega_{\mathrm{pump}}, k_{\mathrm{pump}})$

beat (plasma) wave

$$(\omega_{\mathrm{beat}}, k_{\mathrm{beat}}) = (\omega_{\mathrm{probe}} - \omega_{\mathrm{pump}}, k_{\mathrm{probe}} - k_{\mathrm{pump}})$$

Electromagnetically Induced Transparency (EIT)

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low power probe

$(\omega_{probe}, k_{probe})$

high power pump

$(\omega_{pump}, k_{pump})$

$$\tilde{j}_{probe} = -e\left(n_e v_{probe} + \frac{1}{2}\tilde{n}v_{pump}\right),$$

vanishing of j_{probe} opens window of transparency

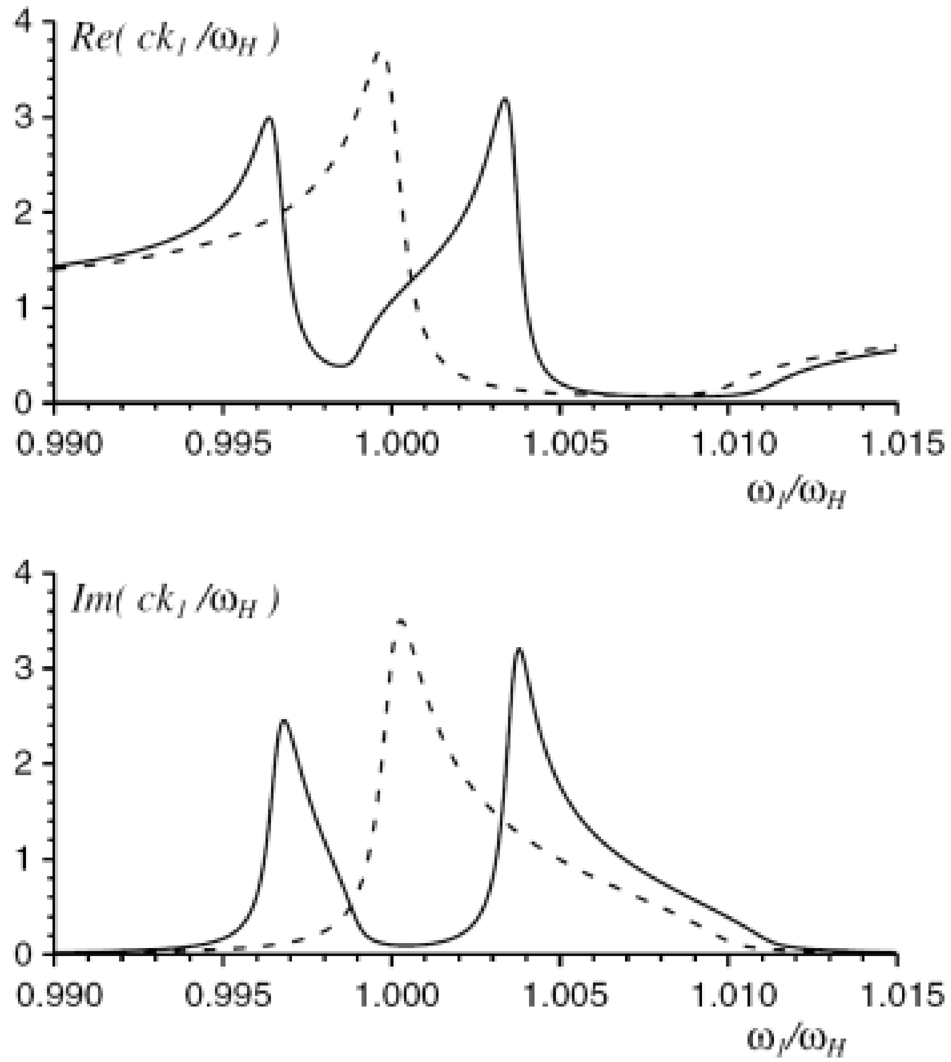
beat (plasma) wave

$(\omega_{beat}, k_{beat}) = (\omega_{probe} - \omega_{pump}, k_{probe} - k_{pump})$

Litvak&Tokman PRL2002

First example of EIT in plasma

- Parallel propagation
  - Right handed pump and probe waves
  - First analysis within cold plasma model
  - Subsequent warm plasma fluid and fully kinetic analyses



(a)

Example: transparency window in cold plasma dispersion with

f_{probe}=100 GHz,

f_{pump}=90GHz with power=250kW/cm^2

$(\omega_{pe}/\omega_{probe})^2$=0.01

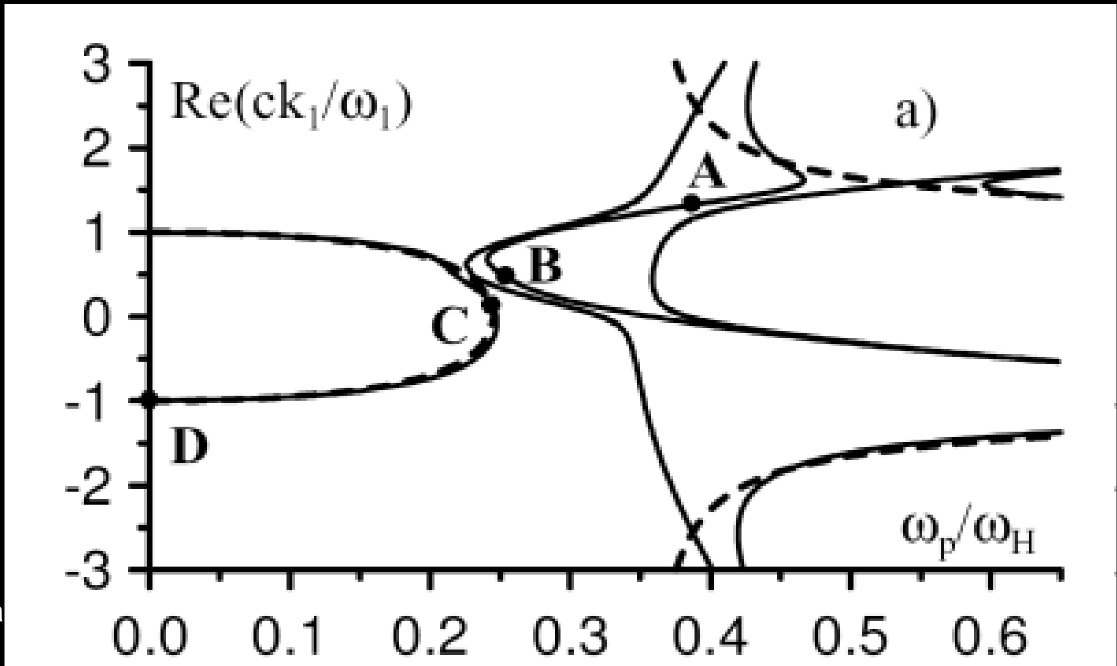
Second example

- Perpendicular propagating X-mode probe
  - Pump is oblique (for beat plasma wave generation) slow X-mode
  - Cold plasma model
  - Transparency window opens in evanescent region between Upper Hybrid resonance and low density cut-off

Kryachko et al. EPS2003

Second example

- Perpendicular propagating X-mode probe
  - Pump is oblique (for beat plasma wave generation) slow X-mode
  - Cold plasma model
  - Transparency window opens in evanescent region between Upper Hybrid resonance and low density cut-off
  - Inhomogeneous plasma: transparency window opens route from slow X (plasma interior) to fast X (escape to vacuum)



Where to go with EIT?

- Open issues in theory (perpendicular X only)
  - A fully kinetic description
  - What is the role of electron-Bernstein waves?

- Develop scenarios for application
  - Extract internal X radiation through transparency window for diagnostics
  - …

Summary

- There is still room for surprises in EC wave theory
- Linear wave propagation
  - Wave energy density and flux
  - Response of wave beams to inhomogeneous absorption
- Quasi-linear plasma response
  - RF-neoclassical synergies
  - Plasma current density evolution in response to ECCD
- Nonlinear plasma-wave interaction
  - Nonlinear absorption important, but to be treated with all complexities of inhomogeneous plasma and wave
  - EIT: expanding theoretical basis and waiting for applications